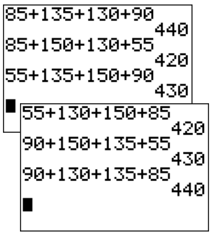
1a Er zijn $3 \cdot 2 \cdot 1 \cdot 1 = 6$ rondritten mogelijk. (het maakt niet uit waar de rondrit begint)
Er zijn steeds twee rondritten met gelijke lengte (maar in omgekeerde volgorde).

1b



```
85+135+130+90
             440
85+150+130+55
             420
55+135+150+90
             430
▮
55+130+150+85
             420
90+150+135+55
             430
90+130+135+85
             440
▮
```

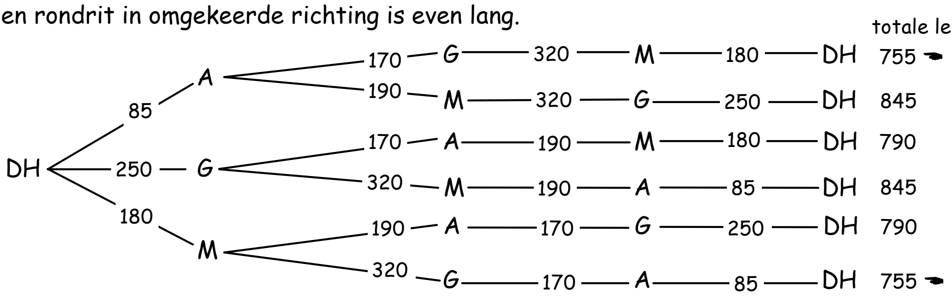
Utrecht-Eindhoven-Zwolle-Haarlem-Utrecht en Utrecht-Haarlem-Zwolle-Eindhoven-Utrecht zijn de kortste.

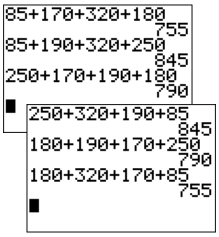
2 De grafen b en d zijn gelijkwaardig.
(beide hebben 5 knooppunten; in 1 knooppunt komen 4 wegen en in elk van de andere 4 knooppunten komen 3 wegen)

3a Er zijn $3 \cdot 2 \cdot 1 \cdot 1 = 6$ rondritten mogelijk.

3b Een rondrit in omgekeerde richting is even lang.

3c

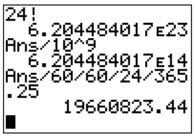




```
85+170+320+180
             755
85+190+320+250
             845
250+170+190+180
             790
▮
250+320+190+85
             845
180+190+170+250
             790
180+320+170+85
             755
▮
```

Advies: DH-A-G-M-DH of omgekeerd DH-M-G-A-DH.

4a Er zijn $24 \cdot 23 \cdot 22 \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 24! \approx 6{,}20 \cdot 10^{23}$ rondritten mogelijk.
(vanuit elk punt kun je op 24 manieren starten, daarna op 23 manieren verder gaan, ...)



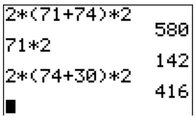
```
24!
  6.204484017ᴇ23
Ans/10^9
  6.204484017ᴇ14
Ans/60/60/24/365
.25
     19660823.44
▮
```

4b Het duurt bijna 20 miljoen jaar. (zie de berekening hiernaast)

5a De (kortste) afstand in km tussen N en R (via V) is $61 + 23 = 84$.
De (kortste) afstand in minuten tussen N en R (via 's H, E en W) is $29 + 21 + 17 + 14 = 81$.

5b De kosten van een treinkaartje enkele reis tweede klas.

6a $Am \rightarrow As$: 2 (2× /week) $\cdot (71 + 74) \cdot 2$ (heen en terug) $= 580$ (km).
$Zw \rightarrow Am$: 1 (1× /week) $\cdot 71 \cdot 2$ (heen en terug) $= 142$ (km).
$Zw \rightarrow Gr$: 2 (2× /week) $\cdot (74 + 30) \cdot 2$ (heen en terug) $= 416$ (km).

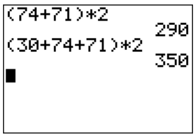


```
2*(71+74)*2
             580
71*2
             142
2*(74+30)*2
             416
▮
```

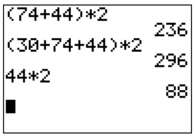
afstand per week van centraal magazijn

$$\text{naar}\ \begin{array}{c|ccccc} & Am & Ap & As & Gr & Zw \\ \hline Am & 0 & 86 & 290 & 350 & 142 \\ Ap & 86 & 0 & 236 & 296 & 88 \\ As & 580 & 472 & 0 & 120 & 296 \\ Gr & 700 & 592 & 120 & 0 & 416 \\ \hline & 1366 & 1025 & 646 & 766 & 942 \end{array} = M$$

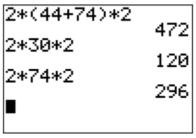
6b Zie de matrix M hiernaast. (heel wat gepuzzel en rekenwerk)



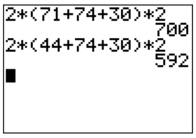
```
(74+71)*2
             290
(30+74+71)*2
             350
▮
```



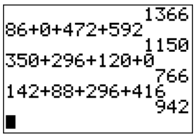
```
(74+44)*2
             236
(30+74+44)*2
             296
44*2
              88
▮
```



```
2*(44+74)*2
             472
2*30*2
             120
2*74*2
             296
▮
```

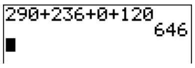


```
2*(71+74+30)*2
             700
2*(44+74+30)*2
             592
▮
```



```
            1366
86+0+472+592
            1150
350+296+120+0
             766
142+88+296+416
             942
▮
```

6c Tel alle afstanden van As naar alle andere vestigingen (kolom onder As) bij elkaar op. Dus $290 + 236 + 0 + 120 = 646$ (km).



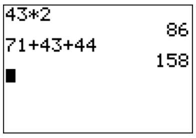
```
290+236+0+120
             646
▮
```

6d Tel in elke kolom de getallen op (zie onder matrix M) $\Rightarrow$ ADVIES: magazijn in Assen (646 km/week).

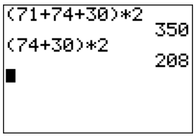
6e $Am \rightarrow Am \rightarrow Ap$: 1 (1× /week) $\cdot (0 + 43) \cdot 2$ (heen en terug) $= 86$ (km).
$Zw \rightarrow Am \rightarrow Ap$: 1 (1× /week) $\cdot (71 + 43 + 44)$ (andere weg terug) $= 158$ (km)
$Am \rightarrow As \rightarrow Gr$: 1 (1× /week) $\cdot (71 + 74 + 30) \cdot 2$ (heen en terug) $= 350$ (km).
$Zw \rightarrow As \rightarrow Gr$: 1 (1× /week) $\cdot (74 + 30) \cdot 2$ (heen en terug) $= 208$ (km)

centraal magazijn

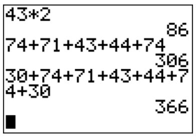
$$\text{naar}\ \begin{array}{c|ccccc} & Am & Ap & As & Gr & Zw \\ \hline Am\text{-}Ap & 86 & 86 & 306 & 366 & 158 \\ As\text{-}Gr & 350 & 296 & 60 & 60 & 208 \\ \hline & 436 & 382 & 366 & 426 & 366 \end{array} = N$$



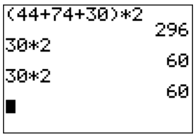
```
43*2
              86
71+43+44
             158
▮
```



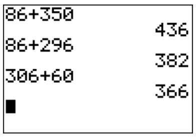
```
(71+74+30)*2
             350
(74+30)*2
             208
▮
```



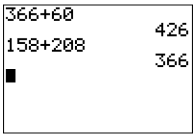
```
43*2
              86
74+71+43+44+74
             306
30+74+71+43+44+7
4+30
             366
▮
```



```
(44+74+30)*2
             296
30*2
              60
30*2
              60
▮
```



```
86+350
             436
86+296
             382
306+60
             366
▮
```



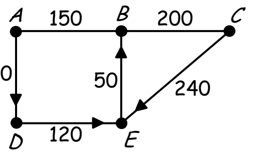
```
366+60
             426
158+208
             366
▮
```

6f Tel in elke kolom de getallen op (zie onder matrix N) $\Rightarrow$ ADVIES: magazijn in Assen of Zwolle (366 km/week).

7a Zie de graaf (punten en verbindingen) hiernaast.

7b "$A \to D$" = 40 en "$D \to A$" = 320.
Zie de verder ingevulde matrix M hiernaast.
(weer even puzzelen)

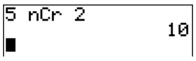
7c In deze matrix is: "$A \to D$" ≠ "$D \to A$".



$$\begin{array}{cc|ccccc|c} & & & & \text{van} & & & \\ & & A & B & C & D & E & \\ & A & 0 & 150 & 350 & 320 & 200 & \\ & B & 150 & 0 & 200 & 170 & 50 & \\ \text{naar} & C & 350 & 200 & 0 & 370 & 250 & = M \\ & D & 40 & 190 & 390 & 0 & 240 & \\ & E & 160 & 310 & 240 & 120 & 0 & \end{array}$$

8a De lijnstukken geven elk een verbinding aan tussen de plaatsen.

8b Telkens twee van de vijf punten kiezen, waarbij de volgorde niet van belang is.
<u>1 1 0 0 0</u> ("1" staat voor "wel" en "0" voor "niet") kan op $\binom{5}{2} = 5\text{nCr}2 = 10$ manieren ⇒ 10 verbindingen.



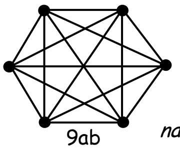
```
5 nCr 2
                10
█
```

8c Er zijn minimaal 4 verbindingen nodig om 5 steden onderling bereikbaar te laten zijn.
(denk de punten achter elkaar aan een snelweg)

9a Verbind elk "knooppunt" met elk "ander knooppunt".
Zie de graaf en de verbindingsmatrix V hiernaast.

9b Een "maximale" graaf heeft $\binom{6}{2} = 6\text{nCr}2 = 15$ wegen.

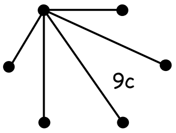
9c Een "minimale" graaf heeft $6 - 1 = 5$ wegen. (zie 2 voorbeelden hiernaast)



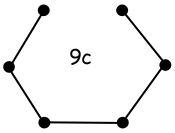
9ab

$$\begin{array}{cc|cccccc|c} & & & & \text{van} & & & & \\ & & A & B & C & D & E & F & \\ & A & 0 & 1 & 1 & 1 & 1 & 1 & \\ & B & 1 & 0 & 1 & 1 & 1 & 1 & \\ \text{naar} & C & 1 & 1 & 0 & 1 & 1 & 1 & = V \\ & D & 1 & 1 & 1 & 0 & 1 & 1 & \\ & E & 1 & 1 & 1 & 1 & 0 & 1 & \\ & F & 1 & 1 & 1 & 1 & 1 & 0 & \end{array}$$

de hoofddiagonaal



9c



9c

10a Een graaf van n knooppunten die maximaal verbonden is heeft $\binom{n}{2}$ wegen.

10b In de verbindingsmatrix van een graaf van n knooppunten die maximaal verbonden is, staan op de hoofddiagonaal nullen en op de andere plaatsen enen.

10c Een graaf van n knooppunten die minimaal verbonden is heeft $n - 1$ wegen.

11a Zie de matrix M hiernaast.

11b De grootste minimale afstand in M is 3 ⇒ de diameter van de graaf in figuur K.14 is 3.

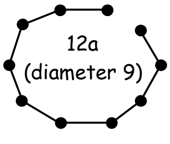
11c Zie de kolomtotalen onder matrix M. (een kleiner totaal betekent een betere bereikbaarheid)
De rangschikking (van bereikbaarheid) is E, A en F, B en D, C.

$$\begin{array}{cc|cccccc|c} & & & & \text{van} & & & & \\ & & A & B & C & D & E & F & \\ & A & 0 & 1 & 2 & 2 & 1 & 2 & \\ & B & 1 & 0 & 3 & 1 & 2 & 2 & \\ \text{naar} & C & 2 & 3 & 0 & 3 & 1 & 2 & = M \\ & D & 2 & 1 & 3 & 0 & 2 & 1 & \\ & E & 1 & 2 & 1 & 2 & 0 & 1 & \\ & F & 2 & 2 & 2 & 1 & 1 & 0 & \\ \hline & & 8 & 9 & 11 & 9 & 7 & 8 & \end{array}$$

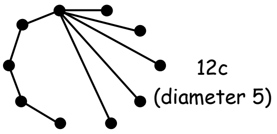
12a De maximale diameter is 9 (zie de graaf hiernaast).

12b Bij maximale verbondenheid is elk punt met elk ander punt verbonden ⇒ de diameter is dan 1.

12c Zie de rechter graaf hiernaast.

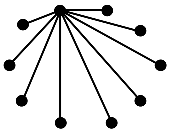


12a (diameter 9)

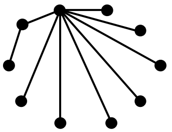


12c (diameter 5)

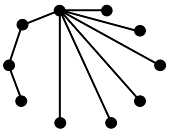
12d De diameter van een graaf met minimale verbondenheid kan 2, 3, 4, 5, 6, 7, 8 of 9 zijn (zie de grafen hier omheen).



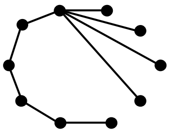
diameter 2



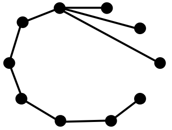
diameter 3



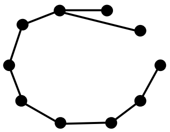
diameter 4



diameter 6

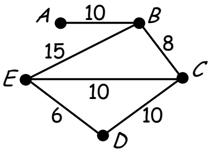


diameter 7

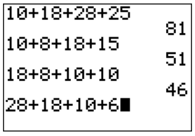


diameter 8

13a Teken eerst de graaf (zie hiernaast) met behulp van de verbindindingsmatrix V en de afstandenmatrix M.
Zet er de getallen van de matrix M bij.
$A \to E = 25$ (gegeven in M) ⇒ $B \to E = 10$.
($A \to E$ via C en D is $10 + 8 + 10 + 6 > 25$)
Nu is de matrix M verder in te vullen (zie hiernaast).



$$\begin{array}{cc|ccccc|cc} & & & & \text{van} & & & & \\ & & A & B & C & D & E & & \text{totaal} \\ & A & 0 & 10 & 18 & 28 & 25 & & 81 \\ & B & 10 & 0 & 8 & 18 & 15 & & 51 \\ \text{naar} & C & 18 & 8 & 0 & 10 & 10 & = M & 46 \\ & D & 28 & 18 & 10 & 0 & 6 & & 62 \\ & E & 25 & 15 & 10 & 6 & 0 & & 56 \end{array}$$



```
10+18+28+25
                81
10+8+18+15
                51
18+8+10+10
                46
28+18+10+6█
```

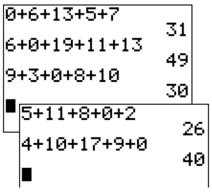
13b Tel nu de getallen per rij in matrix M op. (de totale afstanden van alle punten <u>naar</u> A, <u>naar</u> B enz.)
De totale afstand <u>naar</u> C is het kleinst ⇒ het ziekenhuis komt in C.

14a **verbindingsmatrix**

$$\begin{array}{cc|ccccc|c} & & & & \text{van} & & & \\ & & A & B & C & D & E & \\ & A & 0 & 1 & 0 & 1 & 0 & \\ & B & 1 & 0 & 0 & 0 & 0 & \\ \text{naar} & C & 0 & 1 & 0 & 1 & 0 & = V \\ & D & 1 & 0 & 1 & 0 & 1 & \\ & E & 1 & 0 & 0 & 0 & 0 & \end{array}$$

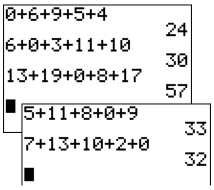
14b **afstandmatrix**

$$\begin{array}{cc|ccccc|cc} & & & & \text{van} & & & & \\ & & A & B & C & D & E & & \\ & A & 0 & 6 & 13 & 5 & 7 & & 31 \\ & B & 6 & 0 & 19 & 11 & 13 & & 49 \\ \text{naar} & C & 9 & 3 & 0 & 8 & 10 & = M & 30 \\ & D & 5 & 11 & 8 & 0 & 2 & & 26 \\ & E & 4 & 10 & 17 & 9 & 0 & & 40 \\ \hline & & 24 & 30 & 57 & 33 & 32 & & \end{array}$$



```
0+6+13+5+7
                31
6+0+19+11+13
                49
9+3+0+8+10
                30
█
```

```
5+11+8+0+2
                26
4+10+17+9+0
                40
█
```



```
0+6+9+5+4
                24
6+0+3+11+10
                30
13+19+0+8+17
                57
█
```

```
5+11+8+0+9
                33
7+13+10+2+0
                32
█
```

14c Zie 14b: $6+0+19+11+13$ (heenweg naar B) $+\,6+0+3+11+10$ (terugweg van B) $= 49+30=79$ (km).

14d 31 (heenweg naar A) $+\,24$ (terugweg van A) $=55$ (km). 26 (heenweg naar D) $+\,33$ (terugweg van D) $=59$ (km).
30 (heenweg naar C) $+\,57$ (terugweg van C) $=87$ (km). 40 (heenweg naar E) $+\,32$ (terugweg van E) $=72$ (km). Dus in A.

15a **directe – wegenmatrix**

$$\text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 3 \\ 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 1 & 0 \end{pmatrix} \end{array} = W$$

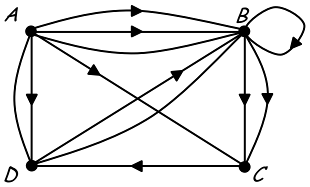
Alle 'niet nullen' in W worden enen in V.
(als er een weg is, is er een verbinding)

15b **verbindingsmatrix**

$$\text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array} = V$$

15c De diameter (het grootste aantal van de minimale stappen tussen elk tweetal punten) van de graaf is 4.
De diameter van de graaf in dit voorbeeld is dus het aantal stappen van D naar A.

16a



16b

$$\text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array} = V$$

verbindingsmatrix

16c Als je in een directe-wegenmatrix alle 'niet nullen' vervangt door 'enen' krijg je de verbindingsmatrix.
Omgekeerd kan niet, want aan een verbinding kun je niet het aantal wegen aflezen.

17a

$$M_1 + M_2 = M_{\text{totaal}} \Rightarrow \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} \text{fil 1} \\ \text{fil 2} \\ \text{fil 3} \end{array} & \begin{pmatrix} 10 & 22 & 18 & 14 \\ 17 & 18 & 12 & 8 \\ 18 & 15 & 16 & 12 \end{pmatrix} \end{array} + \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} \text{fil 1} \\ \text{fil 2} \\ \text{fil 3} \end{array} & \begin{pmatrix} 30 & 18 & 10 & 15 \\ 16 & 29 & 17 & 13 \\ 18 & 21 & 16 & 13 \end{pmatrix} \end{array} = \begin{array}{ccc} & \begin{array}{cccc} A & B & C & D \end{array} & \\ \begin{array}{c} \text{fil 1} \\ \text{fil 2} \\ \text{fil 3} \end{array} & \begin{pmatrix} 40 & 40 & 28 & 29 \\ 33 & 47 & 29 & 21 \\ 36 & 36 & 32 & 25 \end{pmatrix} & \begin{array}{c} 137 \\ 130 \\ 129 \end{array} \\ & \begin{array}{cccc} 109 & 123 & 89 & 75 \end{array} & \end{array}$$

17b $28+29+32=89$. Het totaal aan verkochte wasautomaten C in de drie filialen samen in de eerste helft van 2007.

17c $40+40+28+29=137$. Het totaal aantal verkochte wasautomaten in filiaal 1 in de eerste helft van 2007.

18a A is 2×2-matrix C is 4×1-matrix E is 2×4-matrix
B is 2×3-matrix D is 3×3-matrix F is 1×3-matrix.

18b A en D zijn vierkante matrices.
De som van de getallen op de hoofddiagonaal bij matrix A is $3+0=3$ en bij matrix D is dat $3+2+5=10$.

18c $a_{21}=1$ c_{14} bestaat niet $e_{24}=6$
$b_{21}=4$ $d_{31}=6$ $f_{12}=1$.

19a

$$\begin{array}{cc} & \begin{array}{cccc} \text{L} & \text{LX} & \text{SLX} & \text{D} \end{array} \\ \begin{array}{c} \text{Nob.} \\ \text{Cas.} \end{array} & \begin{pmatrix} 4 & 6 & 3 & 5 \\ 0 & 8 & 4 & 2 \end{pmatrix} \end{array} = V_1$$

19b

$$\begin{array}{cc} & \begin{array}{cccc} \text{L} & \text{LX} & \text{SLX} & \text{D} \end{array} \\ \begin{array}{c} \text{Nob.} \\ \text{Cas.} \end{array} & \begin{pmatrix} 1 & 5 & 8 & 3 \\ 0 & 4 & 8 & 3 \end{pmatrix} \end{array} = V_2$$

19c

$$\begin{array}{cc} & \begin{array}{cccc} \text{L} & \text{LX} & \text{SLX} & \text{D} \end{array} \\ \begin{array}{c} \text{Nob.} \\ \text{Cas.} \end{array} & \begin{pmatrix} 5 & 11 & 11 & 8 \\ 0 & 12 & 12 & 5 \end{pmatrix} \end{array} = V$$

19d

$$\begin{array}{cc} & \begin{array}{cccc} \text{L} & \text{LX} & \text{SLX} & \text{D} \end{array} \\ \begin{array}{c} \text{mei} \\ \text{juni} \end{array} & \begin{pmatrix} 4 & 14 & 7 & 7 \\ 1 & 9 & 16 & 6 \end{pmatrix} \end{array} = T$$

19e $14+9=23$ geeft het totale aantal auto's in de uitvoering LX
dat in mei en juni door beide autobedrijven samen is verkocht.

20a

$$P = \begin{array}{cc} \begin{array}{c} Q \\ T \\ L \\ S \end{array} & \begin{pmatrix} 0{,}8 \\ 1{,}2 \\ 1{,}6 \\ 2{,}0 \end{pmatrix} \end{array} \ (\times €1000).$$

20b Zaak A ontvangt $8\times 0{,}8+6\times 1{,}2+4\times 1{,}6+3\times 2{,}0=26$ ($\times$ €1000).
Zaak B ontvangt $6\times 0{,}8+3\times 1{,}2+0\times 1{,}6+5\times 2{,}0=18{,}4$ ($\times$ €1000).

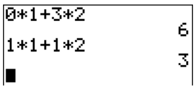
20c

$$I = \begin{array}{cc} \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 26 \\ 18{,}4 \end{pmatrix} \end{array} \ (\times €1000).$$

21a

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = B$$

$$A = \begin{pmatrix} 0 & 3 \\ 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 6 \\ 3 \end{pmatrix} = A \cdot B$$



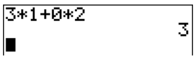
```
0*1+3*2
               6
1*1+1*2
               3
█
```

21b Kan niet.

21c

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = B$$

$$C = \begin{pmatrix} 3 & 0 \end{pmatrix} \qquad (3) = C \cdot B$$

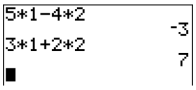


```
3*1+0*2
               3
█
```

21d

$$\begin{pmatrix}1\\2\end{pmatrix}=B$$

$$D=\begin{pmatrix}5&-4\\3&2\end{pmatrix}\quad\begin{pmatrix}-3\\7\end{pmatrix}=D\cdot B$$



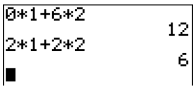
```
5*1-4*2
                 -3
3*1+2*2
                  7
█
```

21e $A\cdot B+D\cdot B=\begin{pmatrix}6\\3\end{pmatrix}+\begin{pmatrix}-3\\7\end{pmatrix}=\begin{pmatrix}3\\10\end{pmatrix}.$

21f

$$\begin{pmatrix}1\\2\end{pmatrix}=B$$

$$2A=\begin{pmatrix}0&6\\2&2\end{pmatrix}\quad\begin{pmatrix}12\\6\end{pmatrix}=2A\cdot B$$



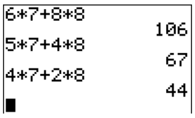
```
0*1+6*2
                 12
2*1+2*2
                  6
█
```

22a $T=\begin{array}{c}A\\B\end{array}\begin{pmatrix}7\\8\end{pmatrix}.$

22b

$$\begin{array}{c}A\\B\end{array}\begin{pmatrix}7\\8\end{pmatrix}$$

$$P=\begin{array}{c|cc} & A & B\\ \hline 3 & 6 & 8\\ 5 & 5 & 4\\ 10 & 4 & 2\end{array}\quad\begin{array}{c}3\\5\\10\end{array}\begin{pmatrix}106\\67\\44\end{pmatrix}=P\cdot T=V$$



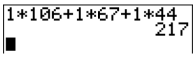
```
6*7+8*8
                106
5*7+4*8
                 67
4*7+2*8
                 44
█
```

De betekenis van de matrix hierboven:
het zijn de aantallen fietsen die per dag gemaakt worden met 3, 5 of 10 versnellingen.

22c

$$\begin{array}{c}3\\5\\10\end{array}\begin{pmatrix}106\\67\\44\end{pmatrix}$$

$$W=\begin{pmatrix}1&1&1\end{pmatrix}\quad(217)=W\cdot V$$



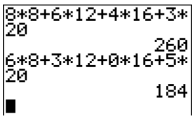
```
1*106+1*67+1*44
                217
█
```

Betekenis:
het aantal fietsen dat per dag gemaakt wordt.

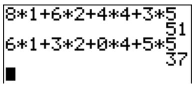
23ab 💻

$$\begin{array}{c|ccc} & \text{prijs} & \text{winst} & \text{software}\\ \hline Q & 8 & 1 & 1\\ T & 12 & 2 & 1\\ L & 16 & 4 & 3\\ S & 20 & 5 & 4\end{array}\;(\times €100)=W$$

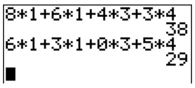
$$V=\begin{array}{c|cccc} & Q & T & L & S\\ \hline A & 8 & 6 & 4 & 3\\ B & 6 & 3 & 0 & 5\end{array}\quad\begin{array}{c|ccc} & \text{prijs} & \text{winst} & \text{software}\\ \hline A & 260 & 51 & 38\\ B & 184 & 37 & 29\end{array}\;(\times €100)=K$$



```
8*8+6*12+4*16+3*
20
                260
6*8+3*12+0*16+5*
20
                184
█
```



```
8*1+6*2+4*4+3*5
                 51
6*1+3*2+0*4+5*5
                 37
█
```

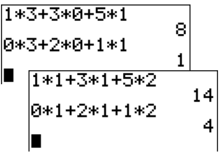


```
8*1+6*1+4*3+3*4
                 38
6*1+3*1+0*3+5*4
                 29
█
```

24a 💻

$$\begin{pmatrix}3&1\\0&1\\1&2\end{pmatrix}=Q$$

$$P=\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}\quad\begin{pmatrix}8&14\\1&4\end{pmatrix}=P\cdot Q$$



```
1*3+3*0+5*1
                  8
0*3+2*0+1*1
                  1
█
```

```
1*1+3*1+5*2
                 14
0*1+2*1+1*2
                  4
█
```

24b 💻

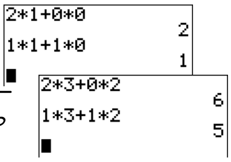
$$\begin{pmatrix}2&0\\1&1\end{pmatrix}=R$$

$$P=\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}\quad P\cdot R\text{ kan niet}$$

24c 💻

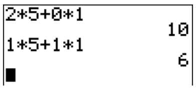
$$\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}=P$$

$$R=\begin{pmatrix}2&0\\1&1\end{pmatrix}\quad\begin{pmatrix}2&6&10\\1&5&6\end{pmatrix}=R\cdot P$$



```
2*1+0*0
                  2
1*1+1*0
                  1
█
```

```
2*3+0*2
                  6
1*3+1*2
                  5
█
```

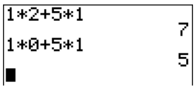


```
2*5+0*1
                 10
1*5+1*1
                  6
█
```

24d 💻

$$\begin{pmatrix}2&0\\1&1\end{pmatrix}=R$$

$$S=\begin{pmatrix}1&5\end{pmatrix}\quad\begin{pmatrix}7&5\end{pmatrix}=S\cdot R$$

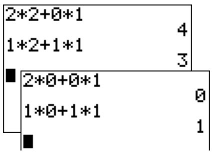


```
1*2+5*1
                  7
1*0+5*1
                  5
█
```

24e 💻

$$\begin{pmatrix}2&0\\1&1\end{pmatrix}=R$$

$$R=\begin{pmatrix}2&0\\1&1\end{pmatrix}\quad\begin{pmatrix}4&0\\3&1\end{pmatrix}=R\cdot R$$



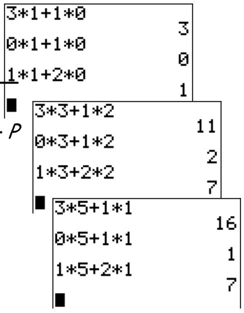
```
2*2+0*1
                  4
1*2+1*1
                  3
█
```

```
2*0+0*1
                  0
1*0+1*1
                  1
█
```

24f 💻

$$\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}=P$$

$$Q=\begin{pmatrix}3&1\\0&1\\1&2\end{pmatrix}\quad\begin{pmatrix}3&11&16\\0&2&1\\1&7&7\end{pmatrix}=Q\cdot P$$



```
3*1+1*0
                  3
0*1+1*0
                  0
1*1+2*0
                  1
█
```

```
3*3+1*2
                 11
0*3+1*2
                  2
1*3+2*2
                  7
█
```

```
3*5+1*1
                 16
0*5+1*1
                  1
1*5+2*1
                  7
█
```

25ab 💻

$$\begin{array}{c|cc} & \text{inkoop} & \text{verkoop}\\ \hline A & 400 & 500\\ B & 275 & 400\\ C & 600 & 800\\ D & 900 & 1200\end{array}=P$$

$$V=\begin{array}{c|cccc} & A & B & C & D\\ \hline \text{filiaal }1 & 10 & 12 & 8 & 4\\ \text{filiaal }2 & 7 & 8 & 5 & 2\\ \text{filiaal }3 & 8 & 8 & 6 & 2\end{array}\quad\begin{array}{c|cc} & \text{inkoop} & \text{verkoop}\\ \hline \text{filiaal }1 & 15700 & 21000\\ \text{filiaal }2 & 9800 & 13100\\ \text{filiaal }3 & 10800 & 14400\end{array}=V\cdot P=T$$

t_{21} geeft het totale inkoopbedrag van filiaal 2.

25cd 💻

$$\begin{array}{c|cc} & \text{inkoop} & \text{verkoop}\\ \hline \text{filiaal }1 & 15700 & 21000\\ \text{filiaal }2 & 9800 & 13100\\ \text{filiaal }3 & 10800 & 14400\end{array}=T$$

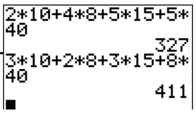
$$Q=\begin{array}{ccc}\text{f1} & \text{f2} & \text{f3}\\ (1 & 1 & 1)\end{array}\quad\begin{array}{cc}\text{inkoop} & \text{verkoop}\\ (36300 & 48500)\end{array}=Q\cdot T$$

De totale winst is $48\,500-36\,300=12\,200$ (€).

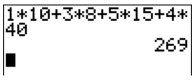
26ac

$$\begin{array}{c|c} & \text{kosten}\\ \hline P & 10\\ Q & 8\\ R & 15\\ T & 40\end{array}=K$$

$$M=\begin{array}{c|cccc} & P & Q & R & T\\ \hline A & 2 & 4 & 5 & 5\\ B & 3 & 2 & 3 & 8\\ C & 1 & 3 & 5 & 4\end{array}\quad\begin{array}{c|c} & \text{kosten}\\ \hline A & 327\\ B & 411\\ C & 269\end{array}=M\cdot K$$



```
2*10+4*8+5*15+5*
40
                327
3*10+2*8+3*15+8*
40
                411
█
```



```
1*10+3*8+5*15+4*
40
                269
█
```

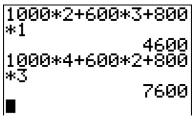
$M\cdot K$ geeft de productiekosten van A, B en C per eenheid.

26bd

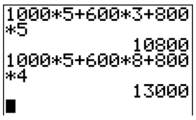
$$\begin{array}{c|cccc} & P & Q & R & T\\ \hline A & 2 & 4 & 5 & 5\\ B & 3 & 2 & 3 & 8\\ C & 1 & 3 & 5 & 4\end{array}=M$$

$$N=\begin{array}{cccc} & A & B & C\\ \text{best.}(& 1000 & 600 & 800)\end{array}\quad\begin{array}{ccccc} & P & Q & R & T\\ \text{best.}(& 4600 & 7600 & 10800 & 13000)\end{array}=N\cdot M$$

$N\cdot M$ geeft de benodigde hoeveelheid grondstoffen en arbeidstijd die nodig is voor de bestelling.

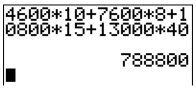


```
1000*2+600*3+800
*1
               4600
1000*4+600*2+800
*3
               7600
█
```



```
1000*5+600*3+800
*5
              10800
1000*5+600*8+800
*4
              13000
█
```

26e $(N\cdot M)\cdot K=\text{best.}\begin{array}{cccc} P & Q & R & T\\ \end{array}\begin{pmatrix}4600 & 7600 & 10800 & 13000\end{pmatrix}\cdot\begin{array}{c|c} & \text{kosten}\\ P & 10\\ Q & 8\\ R & 15\\ T & 40\end{array}=\text{best.}\begin{array}{c}\text{kosten}\\ (788800)\end{array}.$


```
4600*10+7600*8+1
0800*15+13000*40
             788800
█
```

De productiekosten van de totale bestelling zijn 788800 (€).

27a $A=\begin{array}{c|c} & 7\\ 4 & 0\\ 5 & 0\\ 6 & 0\\ 7 & 1\\ 8 & 0\end{array}.$

27b $B=\begin{array}{c|c} & \text{vold.}\\ 4 & 0\\ 5 & 0\\ 6 & 1\\ 7 & 1\\ 8 & 1\end{array}.$

27c $C=\begin{array}{c|c} & \text{tot.}\\ 4 & 1\\ 5 & 1\\ 6 & 1\\ 7 & 1\\ 8 & 1\end{array}.$

27d $D=\begin{array}{c|c} & \text{pnt.}\\ 4 & 4\\ 5 & 5\\ 6 & 6\\ 7 & 7\\ 8 & 8\end{array}.$

27e $E=\text{aant.}\begin{array}{ccc}\text{5A1} & \text{5A2} & \text{5A3}\\ (1 & 1 & 1)\end{array}.$

28a $\begin{pmatrix}a & b & c\\ d & e & f\\ g & h & i\end{pmatrix}=M$

$P=\begin{pmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{pmatrix}\ \begin{pmatrix}3a & 3b & 3c\\ 3d & 3e & 3f\\ 3g & 3h & 3i\end{pmatrix}=P\cdot M=3M$

28b $\begin{pmatrix}a & b & c\\ d & e & f\\ g & h & i\end{pmatrix}=M$

$Q=\begin{pmatrix}1 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 0\end{pmatrix}\ \begin{pmatrix}a & b & c\\ 3d & 3e & 3f\\ 0 & 0 & 0\end{pmatrix}=Q\cdot M$

28c $\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 2 & 0\end{pmatrix}=R$

$M=\begin{pmatrix}a & b & c\\ d & e & f\\ g & h & i\end{pmatrix}\ \begin{pmatrix}a+b & 2c & b\\ d+e & 2f & e\\ g+h & 2i & h\end{pmatrix}=M\cdot R$

29a $\begin{pmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{pmatrix}=A$

$M=\begin{pmatrix}a & b & c & d & e\\ f & g & h & i & j\end{pmatrix}\ \begin{pmatrix}a & b & c & d & e\\ f & g & h & i & j\end{pmatrix}=M\cdot A=M$

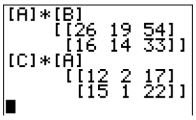
29b $\begin{pmatrix}a & b & c & d & e\\ f & g & h & i & j\end{pmatrix}=M$

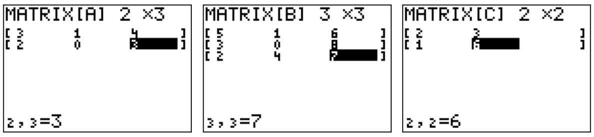
$B=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\ \begin{pmatrix}a & b & c & d & e\\ f & g & h & i & j\end{pmatrix}=B\cdot M=M$

✲✲✲ 🖩 **Neem GR - practicum 13 door.** (zie aan het eind van deze uitwerkingen)

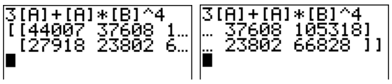
30a $P\cdot Q=\begin{pmatrix}26 & 19 & 54\\ 16 & 14 & 33\end{pmatrix}.$

30b $R\cdot P=\begin{pmatrix}12 & 2 & 17\\ 15 & 1 & 22\end{pmatrix}.$


```
[A]*[B]
      [[26 19 54]
       [16 14 33]]
[C]*[A]
      [[12 2 17]
       [15 1 22]]
█
```

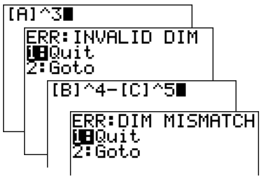

```
MATRIX[A] 2 ×3     MATRIX[B] 3 ×3     MATRIX[C] 2 ×2
[ 3   1   4   ]    [ 5   1   6   ]    [ 2   3       ]
[ 2   0   3   ]    [ 3   0   8   ]    [ 1   6       ]
                   [ 2   4   7   ]

2,3=3              3,3=7              2,2=6
```

30c $3P+P\cdot Q^4=\begin{pmatrix}44007 & 37608 & 105318\\ 27918 & 23802 & 66828\end{pmatrix}.$

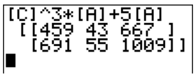

```
3[A]+[A]*[B]^4          3[A]+[A]*[B]^4
[[44007 37608 1…        … 37608 105318]
 [27918 23802 6…        … 23802 66828 ]]
█                       █
```

30d P^3 geeft een foutmelding, want P is geen vierkante matrix.

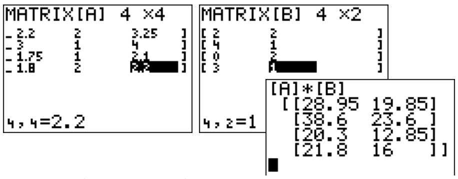
30e $Q^4-R^5=$ geeft een foutmelding want Q^4 en R^5 hebben niet dezelfde afmeting.


```
[A]^3█
ERR:INVALID DIM
1:Quit
2:Goto
[B]^4-[C]^5█
ERR:DIM MISMATCH
1:Quit
2:Goto
```

30f $R^3\cdot P+5P=\begin{pmatrix}459 & 43 & 667\\ 691 & 55 & 1009\end{pmatrix}.$


```
[C]^3*[A]+5[A]
  [[459 43 667 ]
   [691 55 1009]]
█
```

31a $T\cdot G=\begin{array}{r|cccc} & \text{volw.} & \text{kind} & \text{dier} & \text{tent}\\ \text{camping 1} & 5{,}20 & 2{,}20 & 2{,}00 & 3{,}25\\ \text{camping 2} & 7{,}30 & 3{,}00 & 1{,}00 & 4{,}00\\ \text{camping 3} & 3{,}50 & 1{,}75 & 1{,}00 & 2{,}10\\ \text{camping 4} & 4{,}00 & 1{,}80 & 2{,}00 & 2{,}20\end{array}\cdot\begin{array}{r|cc} & V & E\\ \text{volw.} & 2 & 2\\ \text{kind} & 4 & 1\\ \text{dier} & 0 & 2\\ \text{tent} & 3 & 1\end{array}=\begin{array}{r|cc} & V & E\\ \text{camping 1} & 28{,}95 & 19{,}85\\ \text{camping 2} & 38{,}60 & 23{,}60\\ \text{camping 3} & 20{,}30 & 12{,}85\\ \text{camping 4} & 21{,}80 & 16\end{array}=K.$


```
MATRIX[A] 4 ×4          MATRIX[B] 4 ×2
_ 2.2   2   3.25 ]      [ 2   2      ]
_ 3     1   4    ]      [ 4   1      ]
_ 1.75  1   2.1  ]      [ 0   2      ]
_ 1.8   2   2.2  ]      [ 3   1      ]

4,4=2.2                 4,2=1
[A]*[B]
 [[28.95 19.85]
  [38.6  23.6 ]
  [20.3  12.85]
  [21.8  16   ]]
█
```

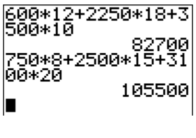
31b $k_{21}=38{,}60$ is het bedrag dat de familie Vrielink (met 2 volw, 4 kinderen en 3 tenten) per dag betaalt op camping 2.

31c Camping 3 was zowel voor familie Vrielink als voor familie Eijssink het voordeligst.

31d $(7\ \ 7\ \ 7\ \ 7)\cdot K=F$ geeft voor beide families de bedragen als ze op elke camping één week hebben gestaan.

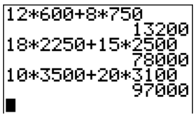
32a De elementen e_{12} en e_{21} van $P\cdot V$ hebben geen betekenis.
(je vermenigvuldigt dan de voorraad van het ene merk met de prijzen van het andere merk)

32b $P\cdot V=\begin{array}{c|ccc} & \text{breedb.} & \text{plasma} & \text{led}\\ A & 600 & 2250 & 3500\\ B & 750 & 2500 & 3100\end{array}\cdot\begin{array}{r|cc} & A & B\\ \text{breedb.} & 12 & 8\\ \text{plasma} & 18 & 15\\ \text{led} & 10 & 20\end{array}=\begin{array}{c|cc} & A & B\\ A & 82700 & \ldots\\ B & \ldots & 105500\end{array}.$


```
600*12+2250*18+3
500*10
             82700
750*8+2500*15+31
00*20
            105500
█
```

e_{11} is de waarde (€) van de TV's van merk A die in voorraad zijn.
e_{22} is de waarde (€) van de TV's van merk B die in voorraad zijn.

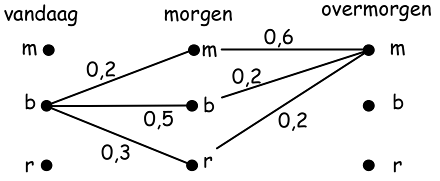
32c
$$V \cdot P = \begin{array}{rc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{r} \text{breedb.} \\ \text{plasma} \\ \text{led} \end{array} & \begin{pmatrix} 12 & 8 \\ 18 & 15 \\ 10 & 20 \end{pmatrix} \end{array} \cdot \begin{array}{rc} & \begin{array}{ccc} \text{breedb.} & \text{plasma} & \text{led} \end{array} \\ \begin{array}{r} A \\ B \end{array} & \begin{pmatrix} 600 & 2250 & 3500 \\ 750 & 2500 & 3100 \end{pmatrix} \end{array} = \begin{array}{rc} & \begin{array}{ccc} \text{breedb.} & \text{plasma} & \text{led} \end{array} \\ \begin{array}{r} \text{breedb.} \\ \text{plasma} \\ \text{led} \end{array} & \begin{pmatrix} 13200 & \dots & \dots \\ \dots & 78000 & \dots \\ \dots & \dots & 97000 \end{pmatrix} \end{array}.$$



```
12*600+8*750
                13200
18*2250+15*2500
                78000
10*3500+20*3100
                97000
▮
```

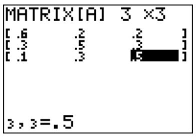
Alleen de elementen op de hoofddiagonaal van $V \cdot P$ hebben betekenis. (de opbrengst bij verkoop van de totale voorraad)
(bij de andere elementen vermenigvuldig je bijvoorbeeld de prijs van breedbeeld A met de voorraad van plasma A)

33 P(overmorgen mooi weer onder de voorwaarde dat het vandaag bewolkt is)
$= P(\text{overmorgen mooi weer} \mid \text{vandaag bewolkt})$
$= P(\text{morgen mooi weer en overmorgen mooi weer} \mid \text{vandaag bewolkt})$
$+ P(\text{morgen bewolkt en overmorgen mooi weer} \mid \text{vandaag bewolkt})$
$+ P(\text{morgen regen en overmorgen mooi weer} \mid \text{vandaag bewolkt})$
$= 0{,}2 \cdot 0{,}6 + 0{,}5 \cdot 0{,}2 + 0{,}3 \cdot 0{,}2 = 0{,}12 + 0{,}10 + 0{,}06 = 0{,}28.$

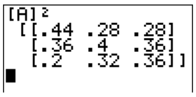


34a De som van de getallen in een kolom is de som van de kansen op alle mogelijke volgende weertypen.
De kansen in een rij hebben niets met elkaar te maken.

34b
$$W^2 = \text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{ccc} m & b & r \end{array} \end{array} \\ \begin{array}{r} m \\ b \\ r \end{array} & \begin{pmatrix} 0{,}44 & 0{,}28 & 0{,}28 \\ 0{,}36 & 0{,}40 & 0{,}36 \\ 0{,}20 & 0{,}32 & 0{,}36 \end{pmatrix} \end{array}.$$



```
MATRIX[A]  3 ×3
[ .6      .2      .2     ]
[ .3      .5      .3     ]
[ .1      .3      .5     ]

3,3=.5
```

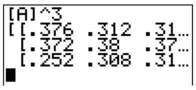


```
[A]²
 [[.44 .28 .28]
  [.36 .4  .36]
  [.2  .32 .36]]
▮
```

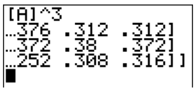
34c 0,20 van W^2 is de kans dat als het op een dag in mei mooi weer is, het twee dagen later zal regenen.

34d Dit is element w_{13} van W^2. Deze kans is 0,28.

34e
$$W^3 = \text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{ccc} m & b & r \end{array} \end{array} \\ \begin{array}{r} m \\ b \\ r \end{array} & \begin{pmatrix} 0{,}376 & 0{,}312 & 0{,}312 \\ 0{,}372 & 0{,}380 & 0{,}372 \\ 0{,}252 & 0{,}308 & 0{,}316 \end{pmatrix} \end{array}.$$



```
[A]^3
[[.376 .312 .31…
 [.372 .38  .37…
 [.252 .308 .31…
▮
```



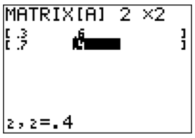
```
[A]^3
….376 .312 .312]
….372 .38  .372]
….252 .308 .316]]
▮
```

0,372 van W^3 is de kans dat als het op een dag in mei regent, het drie dagen later bewolkt zal zijn.

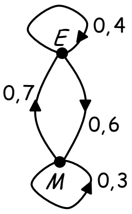
34f 6 mei is 3 dagen na 3 mei. De gevraagde kans wordt gegeven door element w_{33} van W^3 en is 0,316.

34g Voor Egypte zullen de getallen op de eerste rij vrijwel gelijk aan 1 zijn en de andere getallen vrijwel nul.

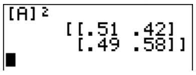
35a Zie de graaf hiernaast.
$$P = \text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{cc} M & E \end{array} \end{array} \\ \begin{array}{r} M \\ E \end{array} & \begin{pmatrix} 0{,}3 & 0{,}6 \\ 0{,}7 & 0{,}4 \end{pmatrix} \end{array}.$$



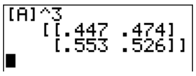
```
MATRIX[A]  2 ×2
[ .3      .6     ]
[ .7      .4     ]

2,2=.4
```



35b
$$P^2 = \text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{cc} M & E \end{array} \end{array} \\ \begin{array}{r} M \\ E \end{array} & \begin{pmatrix} 0{,}51 & 0{,}42 \\ 0{,}49 & 0{,}58 \end{pmatrix} \end{array} \text{ en } P^3 = \text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{cc} M & E \end{array} \end{array} \\ \begin{array}{r} M \\ E \end{array} & \begin{pmatrix} 0{,}447 & 0{,}474 \\ 0{,}553 & 0{,}526 \end{pmatrix} \end{array}.$$



```
[A]²
      [[.51 .42]
       [.49 .58]]
▮
```

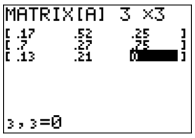


```
[A]^3
   [[.447 .474]
    [.553 .526]]
▮
```

35c De kansen voor hoofdstuk 2 staan nu in P.
De kansen voor hoofdstuk 3 staan in P^2. De gevraagde kans (van M naar E) is 0,49.

35d De kansen voor h4 staan nu in P, die voor h5 in P^2 en voor h6 in P^3. De gevraagde kans (van E naar M) is 0,474.

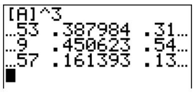
36a
$$\text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{ccc} K & M & S \end{array} \end{array} \\ \begin{array}{r} K \\ M \\ S \end{array} & \begin{pmatrix} 0{,}17 & 0{,}52 & 0{,}25 \\ 0{,}70 & 0{,}27 & 0{,}75 \\ 0{,}13 & 0{,}21 & 0 \end{pmatrix} \end{array} = T.$$



```
MATRIX[A]  3 ×3
[ .17     .52     .25    ]
[ .7      .27     .75    ]
[ .13     .21     0      ]

3,3=0
```

0,27 is de kans dat na een medeklinker weer een medeklinker komt.

36b
$$\text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{ccc} K & M & S \end{array} \end{array} \\ \begin{array}{r} K \\ M \\ S \end{array} & \begin{pmatrix} 0{,}325 & 0{,}388 & 0{,}317 \\ 0{,}534 & 0{,}451 & 0{,}547 \\ 0{,}140 & 0{,}161 & 0{,}136 \end{pmatrix} \end{array} = T^3.$$

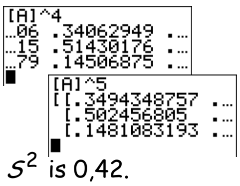


```
[A]^3
…53 .387984 .31…
…9  .450623 .54…
…57 .161393 .13…
▮
```

0,451 is de kans dat drie plaatsen na een medeklinker weer een medeklinker komt.

36c De kans dat vier plaatsen na een medeklinker weer een medeklinker komt, is (ongeveer) 0,514.

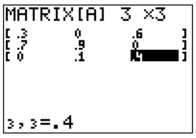
36d De kans dat vijf plaatsen na een klinker een spatie komt, is (ongeveer) 0,148.



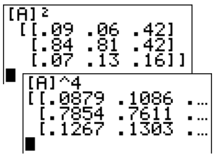
```
[A]^4
…06 .34062949 .…
…15 .51430176 .…
…79 .14506875 .…
▮
```

```
[A]^5
[[.3494348757 .…
 [.502456805  .…
 [.1481083193 .…
▮
```

37a
$$\text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{ccc} B & M & E \end{array} \end{array} \\ \begin{array}{r} B \\ M \\ E \end{array} & \begin{pmatrix} 0{,}3 & 0 & 0{,}6 \\ 0{,}7 & 0{,}9 & 0 \\ 0 & 0{,}1 & 0{,}4 \end{pmatrix} \end{array} = S.$$



```
MATRIX[A]  3 ×3
[ .3      0       .6     ]
[ .7      .9      0      ]
[ 0       .1      .4     ]

3,3=.4
```



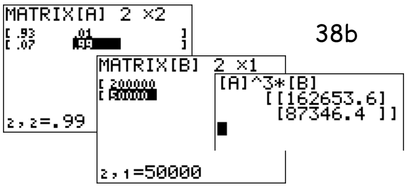
```
[A]²
 [[.09 .06 .42]
  [.84 .81 .42]
  [.07 .13 .16]]
▮
```

```
[A]^4
[[.0879 .1086 .…
 [.7854 .7611 .…
 [.1267 .1303 .…
▮
```

37b Element s_{13} van S^2 is 0,42.
De kans dat twee klanken na een eindklank een beginklank komt.

37c Element s_{12} van S^4 is 0,1086.

38a
$$\text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{cc} A & R \end{array} \end{array} \\ \begin{array}{r} A \\ R \end{array} & \begin{pmatrix} 0{,}93 & 0{,}01 \\ 0{,}07 & 0{,}99 \end{pmatrix} \end{array} = V.$$



```
MATRIX[A]  2 ×2
[ .93     .01    ]
[ .07     .99    ]

2,2=.99
```

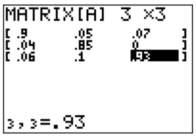
```
MATRIX[B]  2 ×1
[ 200000 ]
[ 50000  ]

2,1=50000
```

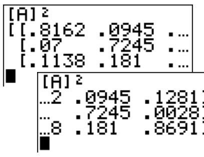
```
[A]^3*[B]
    [[162653.6]
     [87346.4 ]]
▮
```

38b
$$\text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \begin{array}{cc} A & R \end{array} \end{array} \\ \begin{array}{r} A \\ R \end{array} & \begin{pmatrix} 0{,}93 & 0{,}01 \\ 0{,}07 & 0{,}99 \end{pmatrix}^3 \end{array} \cdot \text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \text{inw} \end{array} \\ \begin{array}{r} A \\ R \end{array} & \begin{pmatrix} 200000 \\ 50000 \end{pmatrix} \end{array} \approx \text{naar} \begin{array}{rc} & \begin{array}{c} \text{van} \\ \text{inw} \end{array} \\ \begin{array}{r} A \\ R \end{array} & \begin{pmatrix} 162650 \\ 87350 \end{pmatrix} \end{array}.$$

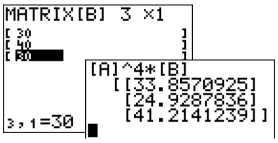
A heeft na 15 jaar 162650 inwoners en R 87350.

39a Bereken met de GR element w_{11} van W^2. Dat is $0{,}8162 \Rightarrow 81{,}6\%$.

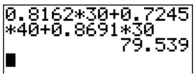

```
MATRIX[A] 3 ×3
[ .9    .05   .07  ]
[ .04   .85   0    ]
[ .06   .1    .93  ]

3,3=.93
```


```
[A]²
[[.8162 .0945 :…
 [.07   .7245 :…
 [.1138 .181  :…
[A]²
…2 .0945 .1281]
…  .7245 .0028]
…8 .181  .8691]]
```

39b $W^4 \cdot \begin{array}{c|c} & \text{van } \% \\ A & 30 \\ \text{naar } B & 40 \\ C & 30 \end{array} \approx \begin{array}{c|c} & \text{van } \% \\ A & 33{,}9 \\ \text{naar } B & 24{,}9 \\ C & 41{,}2 \end{array}$.


```
MATRIX[B] 3 ×1
[ 30   ]
[ 40   ]
[ 30   ]

3,1=30
[A]^4*[B]
  [[33.8570925]
   [24.9287836]
   [41.2141239]]
```

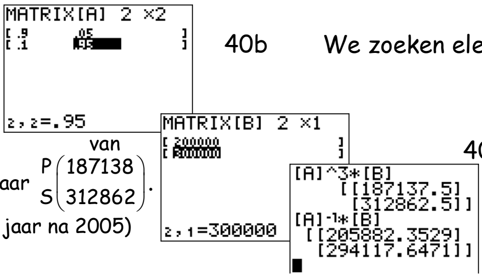
39c Neem de elementen w_{11}, w_{22} en w_{33} van W^2.
Je krijgt $0{,}8162 \cdot 30 + 0{,}7245 \cdot 40 + 0{,}8291 \cdot = 79{,}539$. Dus 79,5%.


```
0.8162*30+0.7245
*40+0.8691*30
           79.539
```

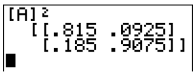
39d De elementen w_{31}, w_{32} en w_{33} van W worden groter.
(de kolomsommen moeten wel 1 blijven)
Zie bijvoorbeeld de matrix W hiernaast.

$$\begin{array}{cc|ccc|} & & & \text{van} & \\ & & A & B & C \\ & A & 0{,}87 & 0{,}04 & 0{,}05 \\ \text{naar} & B & 0{,}03 & 0{,}81 & 0 \\ & C & 0{,}10 & 0{,}15 & 0{,}95 \end{array} = W$$

40a $\begin{array}{cc|cc|} & & \text{van} & \\ & & P & S \\ \text{naar} & P & 0{,}90 & 0{,}05 \\ & S & 0{,}10 & 0{,}95 \end{array} = T$.


```
MATRIX[A] 2 ×2
[ .9   .05  ]
[ .1   .95  ]

2,2=.95
```
```
MATRIX[B] 2 ×1
[ 200000 ]
[ 300000 ]

2,1=300000
```
```
[A]^3*[B]
    [[187137.5]
     [312862.5]]
[A]⁻¹*[B]
  [[205882.3529]
   [294117.6471]]
```

40b We zoeken element t_{22} van $T^2 \Rightarrow 0{,}9075$.

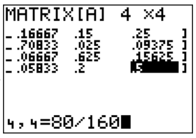

```
[A]²
  [[.815 .0925]
   [.185 .9075]]
```

40c $T^3 \cdot \begin{array}{c|c} & \text{van} \\ \text{naar } P & 200000 \\ S & 300000 \end{array} = \begin{array}{c|c} & \text{van} \\ \text{naar } P & 187138 \\ S & 312862 \end{array}$.
(2020 is 3 perioden van 5 jaar na 2005)

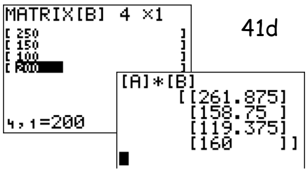
40d $T^{-1} \cdot \begin{array}{c|c} & \text{van} \\ \text{naar } P & 200000 \\ S & 300000 \end{array} = \begin{array}{c|c} & \text{van} \\ \text{naar } P & 205882 \\ S & 294118 \end{array}$.
Dus in 2000 had de stad 294120 inwoners.

41a Deze kans is $\frac{55}{80}$.

41b $\begin{array}{cc|cccc|} & & & \text{van} & & \\ & & N & F & S & E \\ & N & \frac{55}{80} & \frac{20}{120} & \frac{6}{40} & \frac{40}{160} \\ & F & \frac{10}{80} & \frac{85}{120} & \frac{1}{40} & \frac{15}{160} \\ \text{naar} & S & \frac{5}{80} & \frac{8}{120} & \frac{25}{40} & \frac{25}{160} \\ & E & \frac{10}{80} & \frac{7}{120} & \frac{8}{40} & \frac{80}{160} \end{array} \approx \begin{array}{cc|cccc|} & & & \text{van} & & \\ & & N & F & S & E \\ & N & 0{,}688 & 0{,}167 & 0{,}150 & 0{,}250 \\ & F & 0{,}125 & 0{,}708 & 0{,}025 & 0{,}094 \\ \text{naar} & S & 0{,}062 & 0{,}067 & 0{,}625 & 0{,}156 \\ & E & 0{,}125 & 0{,}058 & 0{,}200 & 0{,}500 \end{array} = V$. De kolomsommen moeten 1 zijn.
(op de GR niet afronden)

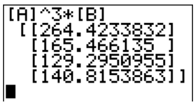

```
MATRIX[A] 4 ×4
….16667  .15   .25    ]
….70833  .025  .09375 ]
….06667  .625  .15625 ]
….05833  .2    .5     ]

4,4=80/160
```

41c $V \cdot \begin{array}{c|c} & \text{van totaal} \\ N & 250 \\ \text{naar } F & 150 \\ S & 100 \\ E & 200 \end{array} \approx \begin{array}{c|c} & \text{van totaal} \\ N & 262 \\ \text{naar } F & 159 \\ S & 119 \\ E & 160 \end{array}$.


```
MATRIX[B] 4 ×1
[ 250 ]
[ 150 ]
[ 100 ]
[ 200 ]

4,1=200
[A]*[B]
    [[261.875]
     [158.75 ]
     [119.375]
     [160    ]]
```

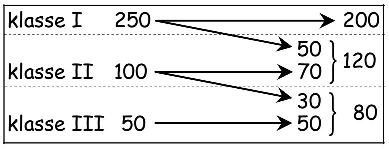
De komende zomer zullen 262 werknemers in Nederland blijven en er zullen 159 werknemers naar Frankrijk gaan.

41d $V^3 \cdot \begin{array}{c|c} & \text{van totaal} \\ N & 250 \\ \text{naar } F & 150 \\ S & 100 \\ E & 200 \end{array} \approx \begin{array}{c|c} & \text{van totaal} \\ N & 264 \\ \text{naar } F & 166 \\ S & 129 \\ E & 161 \end{array}$.

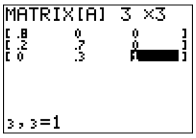

```
[A]^3*[B]
  [[264.4233832]
   [165.466135 ]
   [129.2950955]
   [140.8153863]]
```

Er zullen (over 3 jaar) 129 werknemers naar Spanje gaan.

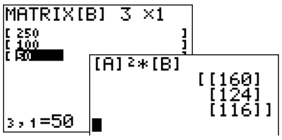
42a $P(\text{boom blijft in I}) = \frac{200}{250} = 0{,}8$.
$P(\text{boom van I naar II}) = \frac{50}{250} = 0{,}2$.
De tabel geeft overgangen hiernaast.



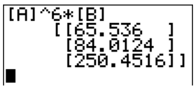
$$\begin{array}{cc|ccc|} & & & \text{van} & \\ & & I & II & III \\ & I & 0{,}8 & 0 & 0 \\ \text{naar} & II & 0{,}2 & 0{,}7 & 0 \\ & III & 0 & 0{,}3 & 1 \end{array} = V$$


```
MATRIX[A] 3 ×3
[ .8   0    0    ]
[ .2   .7   0    ]
[ 0    .3   1    ]

3,3=1
```

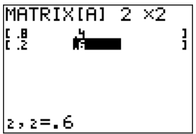
42b $V^2 \cdot \begin{array}{c|c} & \text{van totaal} \\ I & 250 \\ \text{naar } II & 100 \\ III & 50 \end{array} = \begin{array}{c|c} & \text{van totaal} \\ I & 160 \\ \text{naar } II & 124 \\ III & 116 \end{array}$.

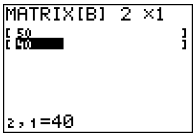

```
MATRIX[B] 3 ×1
[ 250 ]
[ 100 ]
[ 50  ]

3,1=50
[A]²*[B]
       [[160]
        [124]
        [116]]
```

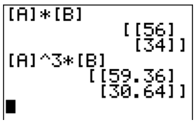
42c $V^6 \cdot \begin{array}{c|c} & \text{van totaal} \\ I & 250 \\ \text{naar } II & 100 \\ III & 50 \end{array} \approx \begin{array}{c|c} & \text{van totaal} \\ I & 66 \\ \text{naar } II & 84 \\ III & 250 \end{array}$.

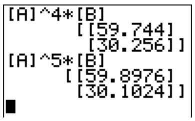

```
[A]^6*[B]
    [[65.536  ]
     [84.0124 ]
     [250.4516]]
```

43a $\begin{array}{cc|cc|} & & \text{van} & \\ & & A & B \\ \text{naar} & A & 0{,}8 & 0{,}4 \\ & B & 0{,}2 & 0{,}6 \end{array} = M$.


```
MATRIX[A] 2 ×2
[ .8   .4  ]
[ .2   .6  ]

2,2=.6
```


```
MATRIX[B] 2 ×1
[ 50 ]
[ 40 ]

2,1=40
```

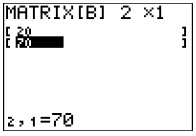

```
[A]*[B]
         [[56]
          [34]]
[A]^3*[B]
      [[59.36]
       [30.64]]
```

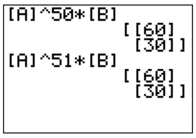

```
[A]^4*[B]
     [[59.744]
      [30.256]]
[A]^5*[B]
    [[59.8976]
     [30.1024]]
```

43b De tweede week: $M \cdot \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 50 \\ B & 40 \end{array} = \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 56 \\ B & 34 \end{array}$; De vierde week: $M^3 \cdot \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 50 \\ B & 40 \end{array} \approx \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 59 \\ B & 31 \end{array}$; De vijfde week: $M^4 \cdot \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 50 \\ B & 40 \end{array} \approx \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 60 \\ B & 30 \end{array}$; De zesde week: $M^5 \cdot \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 50 \\ B & 40 \end{array} \approx \begin{array}{c|c} & \text{van totaal} \\ \text{naar } A & 60 \\ B & 30 \end{array}$.

43c Op den duur zullen de aantallen stabiliseren op 60 mappen A en 30 mappen B.

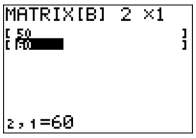
43d Ook nu op den duur 60 mappen A en 30 mappen B. (zie de GR-schermen hiernaast)


```
MATRIX[B] 2 ×1
[ 20 ]
[ 70 ]

2,1=70
```


```
[A]^50*[B]
         [[60]
          [30]]
[A]^51*[B]
         [[60]
          [30]]
```

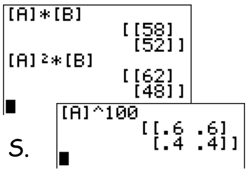
44a $M = \begin{array}{cc|cc|} & & \text{van} & \\ & & L & S \\ \text{naar} & L & 0{,}8 & 0{,}3 \\ & S & 0{,}2 & 0{,}7 \end{array}$ en $B = \begin{array}{cc|c|} & & \text{van totaal} \\ \text{naar} & L & 50 \\ & S & 60 \end{array}$.


```
MATRIX[A] 2 ×2
[ .8   .3  ]
[ .2   .7  ]

2,2=.7
```


```
MATRIX[B] 2 ×1
[ 50 ]
[ 60 ]

2,1=60
```

44a (vervolg)

Over 5 jaar: $\mathcal{M}\cdot B = \text{naar}\ \begin{array}{cc} & \text{van totaal} \\ \begin{matrix}L\\S\end{matrix} & \begin{pmatrix}58\\52\end{pmatrix}\end{array}$ en over 10 jaar: $\mathcal{M}^2\cdot B = \text{naar}\ \begin{array}{cc} & \text{van totaal} \\ \begin{matrix}L\\S\end{matrix} & \begin{pmatrix}62\\48\end{pmatrix}\end{array}$.

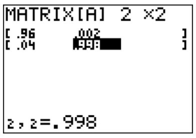


```
[A]*[B]
              [[58]
               [52]]
[A]²*[B]
              [[62]
               [48]]
▮
[A]^100
            [[.6 .6]
             [.4 .4]]
▮
```

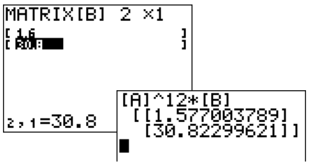
44b Op den duur 60% bij L en 40% bij S. (zie de GR-schermen hiernaast)
Dus op den duur $0{,}6\cdot 110 = 66$ leden bij L en $0{,}4\cdot 110 = 44$ leden bij S.

45ab

$\mathcal{M} = \text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{matrix}N & B\end{matrix} \\ \begin{matrix}N\\B\end{matrix} & \begin{pmatrix}0{,}96 & 0{,}002\\0{,}04 & 0{,}998\end{pmatrix}\end{array}$ en $K = \text{naar}\ \begin{array}{cc} & \text{van totaal} \\ \begin{matrix}N\\B\end{matrix} & \begin{pmatrix}1{,}6\\30{,}8\end{pmatrix}\end{array}$.



```
MATRIX[A] 2 ×2
[ .96      .002    ]
[ .04      .998    ]



2,2=.998
```



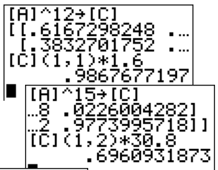
```
MATRIX[B] 2 ×1
[ 1.6              ]
[ 30.8             ]



2,1=30.8
[A]^12*[B]
 [[1.577003789]
  [30.82299621]]
▮
```

Bij 1 januari 2003 (12 maanden verder) hoort $\mathcal{M}^{12}\cdot B \approx \begin{array}{cc} \begin{matrix}N\\B\end{matrix} & \begin{pmatrix}1{,}577\\30{,}823\end{pmatrix}\end{array}$.

Op 1 januari 2003 waren er 1,577 miljard munten in Nederland en 30,823 miljard in het buitenland.

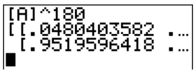
45c Bij 1 januari 2003 hoort $\mathcal{M}^{12}$ met "van N(ederlandse munten) naar N(ederland)" $m_{11}\approx 0{,}6167$.
Op 1 januari 2003 waren er $m_{11}\cdot 1{,}6 \approx 0{,}987$ miljard Nederlandse munten in Nederland.

45d Bij 1 april 2003 hoort $\mathcal{M}^{15}$ met "van B(uitenlandse munten) naar N(ederland)" $m_{12}\approx 0{,}0226$.
Op 1 april 2003 waren er $m_{12}\cdot 30{,}8 \approx 0{,}696$ miljard buitenlandse munten in Nederland.

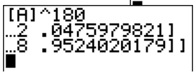


```
[A]^12→[C]
[[.6167298248 .…
 [.3832701752 .…
[C](1,1)*1.6
       .9867677197
▮
[A]^15→[C]
…8 .0226004282]
…2 .9773995718]]
[C](1,2)*30.8
       .6960931873
▮
```

45e Vanaf ongeveer $\mathcal{M}^{180}$ veranderen de machten van $\mathcal{M}$ niet meer.
Dus na ongeveer 180 maanden is de evenwichtssituatie bereikt.



```
[A]^180
[[.0480403582 .…
 [.9519596418 .…
▮
```

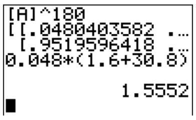


```
[A]^180
…2 .0475979821]
…8 .9524020179]]
▮
```

45f

$\mathcal{M}^{180} \approx \text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{matrix}N & B\end{matrix} \\ \begin{matrix}N\\B\end{matrix} & \begin{pmatrix}0{,}048 & 0{,}048\\0{,}952 & 0{,}952\end{pmatrix}\end{array}$.

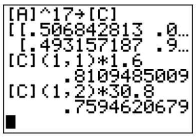
Op den duur zijn er $0{,}048\cdot(1{,}6+30{,}8)\approx 1{,}555$ miljard munten in Nederland in omloop.
Hiervan heeft 4,8% de beeltenis van koningin Beatrix.



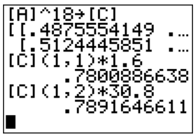
```
[A]^180
[[.0480403582 .…
 [.9519596418 .…
0.048*(1.6+30.8)

            1.5552
▮
```

45g De vraag is: voor welke n in $\mathcal{M}^n$ is $m_{11}\cdot 1{,}6 = m_{12}\cdot 30{,}8$.
Proberen geeft:

$$\left.\begin{array}{l} n=17 \text{ (1 juni 2003)} \Rightarrow \begin{cases} m_{11}\cdot 1{,}6 = 0{,}811 \\ m_{12}\cdot 30{,}8 = 0{,}759 \end{cases} \\ n=18 \text{ (1 juli 2003)} \Rightarrow \begin{cases} m_{11}\cdot 1{,}6 = 0{,}780 \\ m_{12}\cdot 30{,}8 = 0{,}789 \end{cases} \end{array}\right\} \Rightarrow \text{in juni 2003.}$$

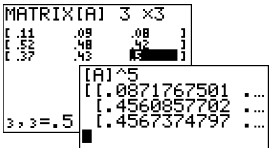


```
[A]^17→[C]
[[.506842813 .0…
 [.493157187 .9…
[C](1,1)*1.6
       .8109485009
[C](1,2)*30.8
       .7594620679
▮
```



```
[A]^18→[C]
[[.4875554149 .…
 [.5124445851 .…
[C](1,1)*1.6
       .7800886638
[C](1,2)*30.8
       .7891646611
▮
```

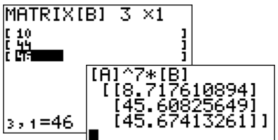
46a Bereken g_{11} van $G^5 \Rightarrow g_{11}\approx 0{,}087$. Dus 8,7% van de inactieven is vijf jaar later nog inactief.



```
MATRIX[A] 3 ×3
[ .11      .09      .08  ]
[ .52      .48      .42  ]
[ .37      .43      .5   ]
[A]^5
[[.0871767501 .…
 [.4560857702 .…
 [.4567374797 .…
3,3=.5
▮
```

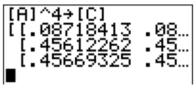
46b

$G^7\cdot \text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{matrix}I & M & N\end{matrix} \\ \begin{matrix}I\\M\\N\end{matrix} & \begin{pmatrix}0{,}11 & 0{,}09 & 0{,}08\\0{,}52 & 0{,}48 & 0{,}42\\0{,}37 & 0{,}43 & 0{,}50\end{pmatrix}\end{array} \cdot \text{naar}\ \begin{array}{cc} & \text{van }\% \\ \begin{matrix}I\\M\\N\end{matrix} & \begin{pmatrix}10\\44\\46\end{pmatrix}\end{array} \approx \text{naar}\ \begin{array}{cc} & \text{van }\% \\ \begin{matrix}I\\M\\N\end{matrix} & \begin{pmatrix}8{,}7\\45{,}6\\45{,}7\end{pmatrix}\end{array}$. Dus 45,7%.

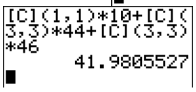


```
MATRIX[B] 3 ×1
[ 10               ]
[ 44               ]
[ 46               ]
[A]^7*[B]
 [[8.717610894]
  [45.60825649]
  [45.67413261]]
3,1=46
▮
```

46c Bereken eerst g_{11}, g_{22} en g_{33}, van G^4.
Je vindt: $g_{11}\cdot 10 + g_{22}\cdot 44 + g_{33}\cdot 46 \approx 42{,}0\%$.

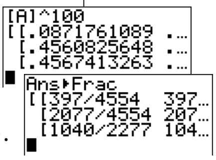


```
[A]^4→[C]
[[.08718413 .08…
 [.45612262 .45…
 [.45669325 .45…
▮
```



```
[C](1,1)*10+[C](
3,3)*44+[C](3,3)
*46
        41.9805527
▮
```

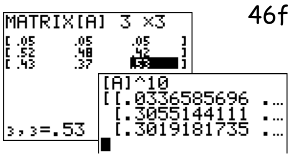
46d Op den duur zijn de drie kolommen van G^n gelijk $\Rightarrow$ stabilisatie.
Je vindt: $g_{11} = g_{12} = g_{12} = \frac{397}{4554}$; $g_{11} = g_{12} = g_{12} = \frac{2077}{4554}$; en $g_{31} = g_{32} = g_{32} = \frac{1040}{2277}$.



```
[A]^100
[[.0871761089 .…
 [.4560825648 .…
 [.4567413263 .…
▮
Ans►Frac
[[397/4554  397…
 [2077/4554 207…
 [1040/2277 104…
▮
```

46e

$G = \text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{matrix}I & M & N\end{matrix} \\ \begin{matrix}I\\M\\N\end{matrix} & \begin{pmatrix}0{,}05 & 0{,}05 & 0{,}05\\0{,}52 & 0{,}48 & 0{,}42\\0{,}43 & 0{,}47 & 0{,}53\end{pmatrix}\end{array}$.



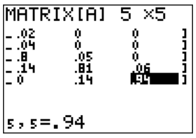
```
MATRIX[A] 3 ×3
[ .05      .05      .05  ]
[ .52      .48      .42  ]
[ .43      .37      .53  ]
[A]^10
[[.0336585696 .…
 [.3055144111 .…
 [.3019181735 .…
3,3=.53
▮
```

46f Het lukt niet met de matrix van 46e.
Met de matrix hieraast lukt het natuurlijk al na 1 jaar.
(dus het is mogelijk)

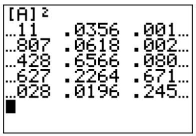
$G = \text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{matrix}I & M & N\end{matrix} \\ \begin{matrix}I\\M\\N\end{matrix} & \begin{pmatrix}0{,}06 & 0{,}06 & 0{,}06\\0{,}40 & 0{,}40 & 0{,}40\\0{,}54 & 0{,}54 & 0{,}54\end{pmatrix}\end{array}$

47a In elke kolom is de som van de elementen onder de hoofddiagonaal groter dan de som van de elementen erboven.

47b Bereken m_{33} van $\mathcal{M}^2 \Rightarrow m_{33} = 0{,}6566$. Dus (ongeveer) 65,7%.



```
MATRIX[A] 5 ×5
_ .02      0        0    ]
_ .04      0        0    ]
_ .8       .05      0    ]
_ .14      .81      .06  ]
_ 0        .14      .94  ]

5,5=.94
```

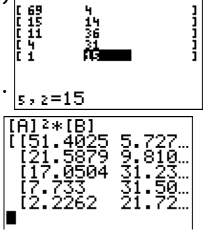


```
[A]²
…11  .0356 .001…
…807 .0618 .002…
…428 .6566 .080…
…627 .2264 .671…
…028 .0196 .245…
▮
```

47c

$\mathcal{M}^2\cdot \text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{matrix}N & A\ (\%)\end{matrix} \\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}69 & 4\\15 & 14\\11 & 36\\4 & 31\\1 & 15\end{pmatrix}\end{array} = \text{naar}\ \begin{array}{cc} & \text{van} \\ & \begin{matrix}N & A\ (\%)\end{matrix} \\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}51{,}4 & 5{,}7\\21{,}6 & 9{,}8\\17{,}1 & 31{,}2\\7{,}7 & 31{,}5\\2{,}{,}2 & 21{,}7\end{pmatrix}\end{array}$.

Dus in Nederland (ongeveer) 7,7%
en in Amsterdam (ongeveer) 31,5%.

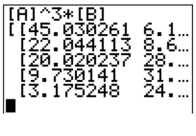


```
MATRIX[B] 5 ×2
[ 69       4        ]
[ 15       14       ]
[ 11       36       ]
[ 4        31       ]
[ 1        15       ]

5,2=15
[A]²*[B]
[[51.4025 5.727…
 [21.5879 9.810…
 [17.0504 31.23…
 [7.733   31.50…
 [2.2262  21.72…
▮
```

47d

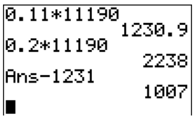
$\mathcal{M}^3\cdot \text{naar}\ \begin{array}{cc} & \text{van} \\ & N\ (\%) \\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}69\\15\\11\\4\\1\end{pmatrix}\end{array} = \text{naar}\ \begin{array}{cc} & \text{van} \\ & N\ (\%) \\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}45{,}0\\22{,}0\\20{,}0\\9{,}7\\3{,}2\end{pmatrix}\end{array}$.

Dus $9{,}7+3{,}2 = 12{,}9\%$.



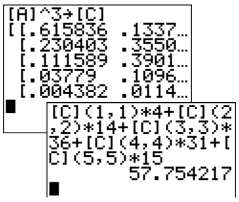
```
[A]^3*[B]
[[45.030261 6.1…
 [22.044113 8.6…
 [20.020237 28.…
 [9.730141  31.…
 [3.175248  24.…
▮
```

47e In 2004 in 11% van de buurten $\Rightarrow 0{,}11 \cdot 11190 \approx 1231$ buurten.
In 2019 in 20,0% (zie 47d) van de buurten $\Rightarrow 0{,}2 \cdot 11190 \approx 2238$ buurten.
Dat zijn $2238 - 1231 = 1007$ buurten meer.



```
0.11*11190
                1230.9
0.2*11190
                  2238
Ans-1231
                  1007
■
```

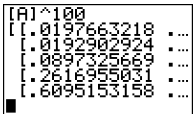
47f Bereken eerst m_{11}, m_{22}, m_{33}, m_{44} en m_{55}, van M^3.
Je vindt: $m_{11} \cdot 4 + m_{22} \cdot 14 + m_{33} \cdot 36 + m_{44} \cdot 31 + m_{55} \cdot 15 \approx 57{,}8\%$ zit in dezelfde klasse.



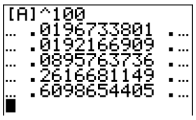
```
[A]^3→[C]
[[.615836 .1337…
 [.230403 .3550…
 [.111589 .3901…
 [.03779  .1096…
 [.004382 .0114…
■
```

```
[C](1,1)*4+[C](2
,2)*14+[C](3,3)*
36+[C](4,4)*31+[
C](5,5)*15
        57.754217
■
```

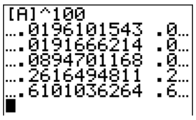
47g Voor grote n verandert M^n niet meer.
Dit betekent bijvoorbeeld dat op den duur in 26,2% van de buurten 25 tot 50 procent allochtoon is en dat in 61,0% van de buurten meer dan 50 procent allochtoon is.



```
[A]^100
[[.0197663218 .…
 [.0192902924 .…
 [.0897325669 .…
 [.2616955031 .…
 [.6095153158 .…
■
```



```
[A]^100
… .0196733801 .…
… .0192166909 .…
… .0895763736 .…
… .2616681149 .…
… .6098654405 .…
■
```



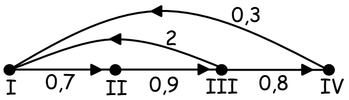
```
[A]^100
….0196101543 .0…
….0191666214 .0…
….0894701168 .0…
….2616494811 .2…
….6101036264 .6…
■
```

48a
- klasse IV is van 3 tot 4 jaar en er is geen klasse V.
- er loopt een pijl van klasse IV naar klasse I en hierbij staat de factor 1000.
- er loopt alleen een pijl van IV naar I en niet van I, II of III naar I.

48b Elke klasse duurt één jaar.

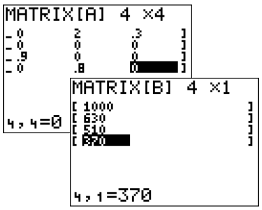
48c Op 1 juni 2007 in III: $0{,}6 \cdot 500 = 300$.
Op 1 juni 2008 in IV: $0{,}8 \cdot 300 = 240$.
Het jaar erna: $240 \cdot 1000 = 240\,000$ nakomelingen.

49a Zie de graaf hiernaast.



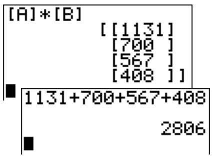
49b

$$L = \text{naar} \begin{array}{cc} & \begin{array}{cccc} & \text{van} & & \\ \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} & \begin{pmatrix} 0 & 0 & 2 & 0{,}3 \\ 0{,}7 & 0 & 0 & 0 \\ 0 & 0{,}9 & 0 & 0 \\ 0 & 0 & 0{,}8 & 0 \end{pmatrix} \end{array}.$$



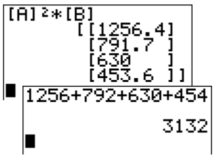
```
MATRIX[A]  4 ×4
[ 0      2      .3     ]
[ 0      0      0      ]
[ .9     0      0      ]
[ 0      .8     0      ]

4,4=0
```

```
MATRIX[B]  4 ×1
[ 1000               ]
[ 630                ]
[ 510                ]
[ 370                ]

4,1=370
```



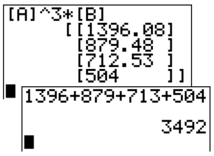
```
[A]*[B]
          [[1131]
            [700 ]
            [567 ]
            [408 ]]
■
```

```
1131+700+567+408
                  2806
■
```



```
[A]²*[B]
        [[1256.4]
          [791.7 ]
          [630   ]
          [453.6 ]]
■
```

```
1256+792+630+454
                  3132
■
```



```
[A]^3*[B]
      [[1396.08]
        [879.48 ]
        [712.53 ]
        [504    ]]
■
```

```
1396+879+713+504
                  3492
■
```

49cd

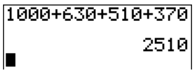
$t = 0$
$$P = \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \\ \end{array} \begin{array}{c} \text{van} \\ \begin{pmatrix} 1000 \\ 630 \\ 510 \\ 370 \end{pmatrix} \\ \hline\hline 2510 \end{array}.$$

$t = 1$
$$L \cdot P = \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \\ \end{array} \begin{array}{c} \text{van} \\ \begin{pmatrix} 1131 \\ 700 \\ 567 \\ 408 \end{pmatrix} \\ \hline\hline 2806 \end{array}.$$

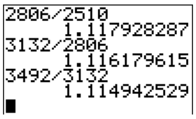
$t = 2$
$$L^2 \cdot P \approx \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \\ \end{array} \begin{array}{c} \text{van} \\ \begin{pmatrix} 1256 \\ 792 \\ 630 \\ 454 \end{pmatrix} \\ \hline\hline 3132 \end{array}.$$

$t = 3$
$$L^3 \cdot P \approx \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \\ \end{array} \begin{array}{c} \text{van} \\ \begin{pmatrix} 1396 \\ 879 \\ 713 \\ 504 \end{pmatrix} \\ \hline\hline 3492 \end{array}.$$

$$2510 \xrightarrow{\times 1{,}118} 2806 \xrightarrow{\times 1{,}116} 3132 \xrightarrow{\times 1{,}115} 3492$$



```
1000+630+510+370
                  2510
■
```

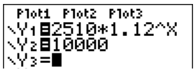


```
2806/2510
      1.117928287
3132/2806
      1.116179615
3492/3132
      1.114942529
■
```

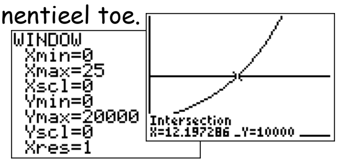
De groeifactoren zijn ongeveer gelijk $\Rightarrow$ de totale populatie neemt bij benadering exponentieel toe.

49e Exponentiële groei $\Rightarrow N = b \cdot g^t$ met $b = 2510$ en $g \approx 1{,}12 \Rightarrow N = 2510 \cdot 1{,}12^t$.

49f Los op: $2510 \cdot 1{,}12^t = 10\,000$ (intersect) $\Rightarrow t \approx 12{,}2$. Dus na ruim 12 jaar.



```
 Plot1  Plot2  Plot3
\Y1■2510*1.12^X
\Y2■10000
\Y3=■
```



```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=0
 Ymin=0
 Ymax=20000
 Yscl=0
 Xres=1
```

```
Intersection
X=12.197286  Y=10000
```

50a Er zijn 7 leeftijdsklassen $\Rightarrow$ de afmeting van de Lesliematrix L is 7×7.

50b Het element l_{21} (betekenis: de kans om van I naar II te gaan) in de Lesliematrix L is zeker niet nul.

50c l_{54} (betekenis: de kans om van IV naar V te gaan) in de Lesliematrix L is zeker niet nul.

50d l_{11} (betekenis: de kans om van I naar I te gaan) in de Lesliematrix L hoeft niet nul te zijn.
Het zou kunnen dat exemplaren uit I voor nakomelingen kunnen zorgen.
De andere elementen op de hoofddiagonaal moeten wel nul zijn omdat de exemplaren na een periode van 5 jaar naar de volgende klasse gaan.

50e De getallen in de eerste rij geven het gemiddelde aantal nakomelingen per individu uit een leeftijdsklasse.

50f De andere getallen (die niet in de eerste rij staan) zijn kansen.

51a l_{32} is de kans om in een periode van 5 jaar uit de tweede in de derde leeftijdsklasse te komen.
l_{12} is het gemiddelde aantal nakomelingen in een periode van 5 jaar per individu uit de tweede leeftijdsklasse.
l_{21} is de kans om in een periode van 5 jaar uit de eerste in de tweede leeftijdsklasse te komen.
l_{11} is het gemiddelde aantal nakomelingen in een periode van 5 jaar per individu uit de eerste leeftijdsklasse.

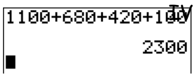
51b

$t = 0$
$$P = \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \\ \end{array} \begin{array}{c} \text{van} \\ \begin{pmatrix} 1100 \\ 680 \\ 420 \\ 100 \end{pmatrix} \\ \hline\hline 2300 \end{array};$$

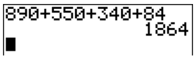
$t = 1$
$$L \cdot P = \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \\ \end{array} \begin{array}{c} \text{van} \\ \begin{pmatrix} 890 \\ 550 \\ 340 \\ 84 \end{pmatrix} \\ \hline\hline 1864 \end{array};$$

$t = 2$
$$L^2 \cdot P = \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \\ \end{array} \begin{array}{c} \text{van} \\ \begin{pmatrix} 720 \\ 445 \\ 275 \\ 68 \end{pmatrix} \\ \hline\hline 1508 \end{array}$$

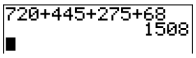
Totaal: 2300 1864 1508



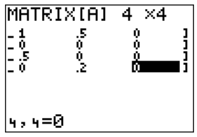
```
1100+680+420+100
                  2300
■
```



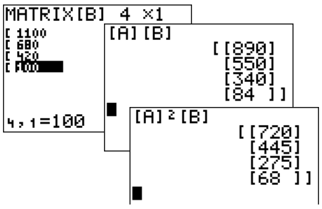
```
890+550+340+84
                  1864
■
```



```
720+445+275+68
                  1508
■
```



```
MATRIX[A]  4 ×4
[ 1      .5     0      ]
[ 0      0      0      ]
[ .5     0      0      ]
[ 0      .2     0      ]

4,4=0
```



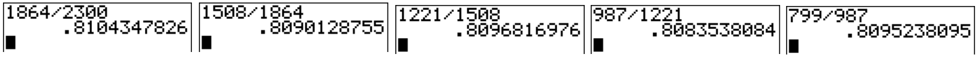
```
MATRIX[B]  4 ×1
[ 1100
[ 680
[ 420
[ 100

4,1=100
```

```
[A][B]
          [[890]
            [550]
            [340]
            [84 ]]
■
```

```
[A]²[B]
          [[720]
            [445]
            [275]
            [68 ]]
■
```

51c Totalen: in het begin: na 5 jaar: na10jaar: na 15 jaar: na 20 jaar: na 25 jaar:

$$2300 \xrightarrow{\times 0,81} 1864 \xrightarrow{\times 0,81} 1508 \xrightarrow{\times 0,81} 1221 \xrightarrow{\times 0,81} 987 \xrightarrow{\times 0,81} 799$$



```
1864/2300
      .8104347826
```

```
1508/1864
      .8090128755
```

```
1221/1508
      .8096816976
```

```
987/1221
      .8083538084
```

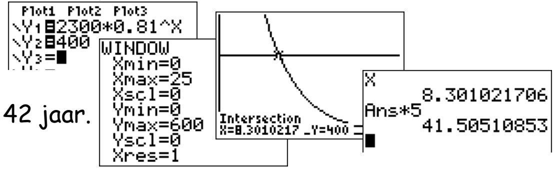
```
799/987
      .8095238095
```

De groeifactoren zijn ongeveer gelijk ⇒ de totale populatie neemt elke 5 jaar met ongeveer 19% af.

51d $N = 2300 \cdot 0,81^t$. (t gemeten in perioden van 5 jaar)

51e $N = 2300 \cdot 0,81^t = 400$ (intersect) $\Rightarrow t \approx 8,3$ (perioden van 5 jaar)

$N = 2300 \cdot 0,81^t < 400$ (zie een plot) ⇒ voor het eerst na (ongeveer) 42 jaar.



```
Plot1 Plot2 Plot3
\Y1=2300*0.81^X
\Y2=400
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=0
 Ymin=0
 Ymax=600
 Yscl=0
 Xres=1
```

```
Intersection
X=8.3010217  Y=400
```

```
X
         8.301021706
Ans*5
         41.50510853
```

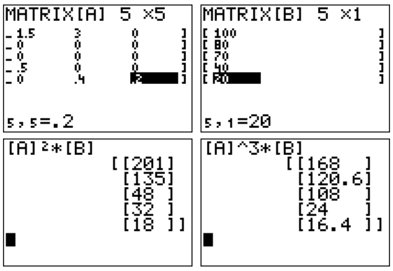
52a Omdat er 4-jarige (klasse ≥ 4) zijn die nog 5 jaar (ook klasse ≥ 4) worden is $l_{55} \neq 0$.
Er is gegeven dat 80% van klasse ≥ 4 binnen één jaar sterft ⇒ $l_{55} = 0,2$.
Haal verder uit beide bevolkingspiramiden:

$P(0 \to 1) = l_{21} = \frac{25+35}{48+52} = \frac{60}{100} = 0,6$; $P(1 \to 2) = l_{32} = \frac{30+34}{38+42} = \frac{64}{80} = 0,8$;

$P(2 \to 3) = l_{43} = \frac{17+18}{35+35} = \frac{35}{70} = 0,5$ en $P(3 \to 4) = l_{54} = \frac{10+10-0,2\cdot(9+11)}{17+23} = \frac{16}{40} = 0,4$.

Verder staat in de tekst: $P(2 \to 0) = l_{13} = 1,5$ en $P(3 \to 0) = l_{14} = 3$.
De andere elementen zijn 0. Hieruit volgt dan de Lesliematrix L hiernaast.

$$L = \text{naar} \begin{array}{c|ccccc} & & & \text{van} & & \\ & 0 & 1 & 2 & 3 & \geq 4 \\ \hline 0 & 0 & 0 & 1,5 & 3 & 0 \\ 1 & 0,6 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0,8 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0,5 & 0 & 0 \\ \geq 4 & 0 & 0 & 0 & 0,4 & 0,2 \end{array}$$



```
MATRIX[A] 5 ×5
 1.5    3     0     ]
 0      0     0     ]
 0      0     0     ]
 .5     0     0     ]
 0      .4    .2    ]

5,5=.2
```

```
MATRIX[B] 5 ×1
[ 100          ]
[ 80           ]
[ 70           ]
[ 40           ]
[ 20           ]

5,1=20
```

```
[A]²*[B]
          [[201]
           [135]
           [48 ]
           [32 ]
           [18 ]]
```

```
[A]^3*[B]
          [[168  ]
           [120.6]
           [108  ]
           [24   ]
           [16.4 ]]
```

52b

$$L^2 \cdot \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 100 \\ 1 & 80 \\ 2 & 70 \\ 3 & 40 \\ \geq 4 & 20 \end{array} \overset{t=2}{=} \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 201 \\ 1 & 135 \\ 2 & 48 \\ 3 & 32 \\ \geq 4 & 18 \end{array} \quad \text{en} \quad L^3 \cdot \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 100 \\ 1 & 80 \\ 2 & 70 \\ 3 & 40 \\ \geq 4 & 20 \end{array} \overset{t=3}{\approx} \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 168 \\ 1 & 121 \\ 2 & 108 \\ 3 & 24 \\ \geq 4 & 16 \end{array}.$$

53ab

$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & 0 & d \\ \text{II} & a & 0 & 0 & 0 \\ \text{III} & 0 & b & 0 & 0 \\ \text{IV} & 0 & 0 & c & 0 \end{array} = L$$

$$L = \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & 0 & d \\ \text{II} & a & 0 & 0 & 0 \\ \text{III} & 0 & b & 0 & 0 \\ \text{IV} & 0 & 0 & c & 0 \end{array} \qquad \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & cd & 0 \\ \text{II} & 0 & 0 & 0 & ad \\ \text{III} & ab & 0 & 0 & 0 \\ \text{IV} & 0 & bc & 0 & 0 \end{array} = L^2$$

$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & cd & 0 \\ \text{II} & 0 & 0 & 0 & ad \\ \text{III} & ab & 0 & 0 & 0 \\ \text{IV} & 0 & bc & 0 & 0 \end{array} = L^2$$

$$L^2 = \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & cd & 0 \\ \text{II} & 0 & 0 & 0 & ad \\ \text{III} & ab & 0 & 0 & 0 \\ \text{IV} & 0 & bc & 0 & 0 \end{array} \qquad \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & abcd & 0 & cd & 0 \\ \text{II} & 0 & abcd & 0 & 0 \\ \text{III} & 0 & 0 & abcd & 0 \\ \text{IV} & 0 & bc & 0 & abcd \end{array} = L^4$$

53b Je kunt in vier stappen alleen van een klasse naar diezelfde klasse. (bestudeer de graaf)

$\text{I} \xrightarrow{a} \text{II} \xrightarrow{b} \text{III} \xrightarrow{c} \text{V} \xrightarrow{d} \text{I} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{abcd} \text{I} \Rightarrow$ in L^4 is $l_{11} = abcd$.

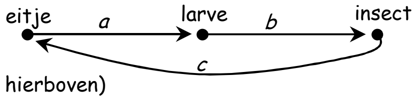
$\text{II} \xrightarrow{b} \text{III} \xrightarrow{c} \text{IV} \xrightarrow{d} \text{I} \xrightarrow{a} \text{II} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{bcda} \text{I} \Rightarrow$ in L^4 is $l_{22} = abcd$.

$\text{III} \xrightarrow{c} \text{IV} \xrightarrow{d} \text{I} \xrightarrow{a} \text{II} \xrightarrow{b} \text{III} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{cdab} \text{I} \Rightarrow$ in L^4 is $l_{33} = abcd$.

$\text{IV} \xrightarrow{d} \text{I} \xrightarrow{a} \text{II} \xrightarrow{b} \text{III} \xrightarrow{c} \text{IV} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{dabc} \text{I} \Rightarrow$ in L^4 is $l_{44} = abcd$.

53c In elke klasse wordt het aantal dieren in vier jaar tijd vermenigvuldigd met $abcd$.

54a Maak eerst de graaf. (zie hierboven)



Maak met behulp van deze graaf de Lesliematrix L. (zie hiernaast)

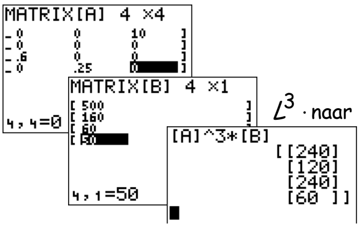
$$L = \text{naar} \begin{array}{c|ccc} & & \text{van} & \\ & \text{e} & \text{l} & \text{i} \\ \hline \text{e} & 0 & 0 & c \\ \text{l} & a & 0 & 0 \\ \text{i} & 0 & b & 0 \end{array} \qquad L^3 = \text{naar} \begin{array}{c|ccc} & & \text{van} & \\ & \text{e} & \text{l} & \text{i} \\ \hline \text{e} & abc & 0 & 0 \\ \text{l} & 0 & abc & 0 \\ \text{i} & 0 & 0 & abc \end{array}$$

54b Maak met behulp van de driestapswegen in de graaf de matrix L^3. (zie hiernaast)

54c $abc = 1$. (L^3 is de eenheidsmatrix) 54d $abc = 1,2$. 54e $abc < 1$.

55a P(pasgeboren dier wordt één jaar) $= 0,8 \cdot 0,6 = 0,48$.

55b Op 1 juli 2008, anderhalf jaar na 1 januari 2007, is $t = 3$.
De samenstelling op $t = 3$ bereken je met $L^3 \cdot P$.
(zie de berekening en het antwoord hiernaast)



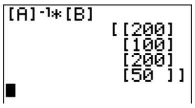
```
MATRIX[A] 4 ×4
 0      0     10    ]
 0      0     0     ]
 .6     0     0     ]
 0      .25   0     ]

4,4=0
```

```
MATRIX[B] 4 ×1
[ 500          ]
[ 160          ]
[ 60           ]
[ 50           ]

4,1=50
```

```
[A]^3*[B]
          [[240]
           [120]
           [240]
           [60 ]]
```

$$L^3 \cdot \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 500 \\ \text{II} & 160 \\ \text{III} & 60 \\ \text{IV} & 50 \end{array} \overset{t=3}{=} \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 240 \\ \text{II} & 120 \\ \text{III} & 240 \\ \text{IV} & 60 \end{array}$$

55c Op 1 juli 2006, een half jaar voor 1 januari 2007, is $t = -1$.
De samenstelling op $t = -1$ bereken je met $L^{-1} \cdot P$.
(zie de berekening en het antwoord hiernaast)



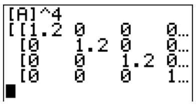
```
[A]⁻¹*[B]
              [[200]
               [100]
               [200]
               [50 ]]
▮
```

t = −1

$$L^{-1}\cdot \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 500 \\ \text{II} & 160 \\ \text{III} & 60 \\ \text{IV} & 50 \end{array} = \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 200 \\ \text{II} & 100 \\ \text{III} & 200 \\ \text{IV} & 50 \end{array}$$

55d

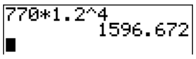
$$L^4 = \text{naar} \begin{array}{c|cccc} & & \text{van} & & \\ & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 1{,}2 & 0 & 0 & 0 \\ \text{II} & 0 & 1{,}2 & 0 & 0 \\ \text{III} & 0 & 0 & 1{,}2 & 0 \\ \text{IV} & 0 & 0 & 0 & 1{,}2 \end{array}.$$



```
[A]^4
[[1.2 0   0   0…
 [0   1.2 0   0…
 [0   0   1.2 0…
 [0   0   0   1…
▮
```

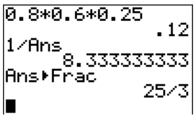
Dit betekent dat de hoeveelheid elke 2 jaar (vier halve jaren) met 20% toeneemt.

55e Elke twee jaar wordt de hoeveelheid met 1,20 vermenigvuldigd.
Op 1-1-2015 (4×2 jaar na 1-1-2007) zijn er $770 \times 1{,}20^4 \approx 1597$ dieren.



```
770*1.2^4
        1596.672
▮
```

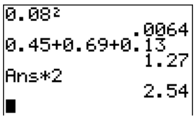
55f Over een periode van twee jaar (4 halve jaren) geen groei $\Rightarrow L^4$ is de eenheidsmatrix.
Er geldt dus $0{,}8 \cdot 0{,}6 \cdot 0{,}25 \cdot v = 1 \Rightarrow 0{,}12v = 1 \Rightarrow v = \frac{25}{3} \approx 8{,}33$.
Vervang nu de 10 in de oorspronkelijk Lesliematrix door $\frac{25}{3}$. (afronden is niet nodig)



```
0.8*0.6*0.25
             .12
1/Ans
      8.333333333
Ans▸Frac
             25/3
▮
```

56a P(twee perioden van 10 jaar 80+) $= 0{,}080 \cdot 0{,}080 = 0{,}0064$.

56b Per individu $0{,}45 + 0{,}69 + 0{,}13 = 1{,}27$ kind $\Rightarrow$ per gezin $2 \cdot 1{,}27 \approx 2{,}5$ kind.



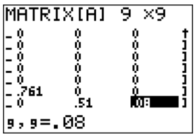
```
0.08²
           .0064
0.45+0.69+0.13
            1.27
Ans*2
            2.54
▮
```

56c 2032 is 50 jaar (5 perioden van 10 jaar) na 1982 $\Rightarrow$ bereken $L^5 \cdot P$.

$$L = \text{naar} \begin{array}{c|ccccccccc} & & & & & \text{van} & & & & \\ & \text{0-9} & \text{10-19} & \text{20-29} & \text{30-39} & \text{40-49} & \text{50-59} & \text{60-69} & \text{70-79} & \text{80+} \\ \hline \text{0-9} & 0 & 0{,}45 & 0{,}69 & 0{,}13 & 0 & 0 & 0 & 0 & 0 \\ \text{10-19} & 0{,}930 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{20-29} & 0 & 0{,}993 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{30-39} & 0 & 0 & 0{,}987 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{40-49} & 0 & 0 & 0 & 0{,}981 & 0 & 0 & 0 & 0 & 0 \\ \text{50-59} & 0 & 0 & 0 & 0 & 0{,}962 & 0 & 0 & 0 & 0 \\ \text{60-69} & 0 & 0 & 0 & 0 & 0 & 0{,}907 & 0 & 0 & 0 \\ \text{70-79} & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}761 & 0 & 0 \\ \text{80+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}510 & 0{,}080 \end{array}$$

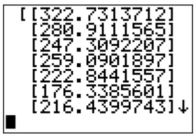
1982 2032

$$\Rightarrow L^5 \cdot \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline \text{0-9} & 205 \\ \text{10-19} & 258 \\ \text{20-29} & 169 \\ \text{30-39} & 127 \\ \text{40-49} & 99 \\ \text{50-59} & 74 \\ \text{60-69} & 48 \\ \text{70-79} & 23 \\ \text{80+} & 5 \\ \hline\hline & 1008 \end{array} \approx \text{naar} \begin{array}{c|c} & \text{van totaal} \\ \hline \text{0-9} & 323 \\ \text{10-19} & 281 \\ \text{20-29} & 247 \\ \text{30-39} & 259 \\ \text{40-49} & 223 \\ \text{50-59} & 176 \\ \text{60-69} & 216 \\ \text{70-79} & 109 \\ \text{80+} & 45 \\ \hline\hline & 1879 \end{array}.$$

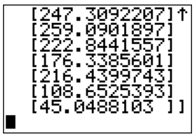


```
MATRIX[A]  9 ×9
 0      0      0    ↑
 0      0      0    ]
 0      0      0    ]
 0      0      0    ]
 0      0      0    ]
 .761   0      0    ]
 0      .51    .08  ]
9,9=.08
```

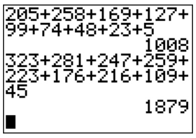
```
MATRIX[B]  9 ×1
[ 169          ↑
[ 127          ]
[ 99           ]
[ 74           ]
[ 48           ]
[ 23           ]
[ 5            ]
9,1=5
```



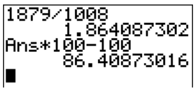
```
 [[322.7313712]
  [280.9111565]
  [247.3092207]
  [259.0901897]
  [222.8441557]
  [176.3385601]
  [216.4399743]↓
▮
```



```
  [247.3092207]↑
  [259.0901897]
  [222.8441557]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
▮
```



```
205+258+169+127+
99+74+48+23+5
            1008
323+281+247+259+
223+176+216+109+
45
            1879
▮
```

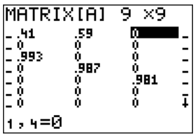


```
1879/1008
     1.864087302
Ans*100-100
     86.40873016
▮
```

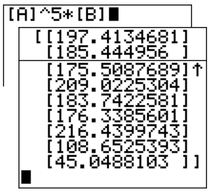
56d

2032

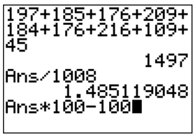
$$(L^*)^5 \cdot P \approx \begin{array}{c} \begin{pmatrix} 197 \\ 185 \\ 176 \\ 209 \\ 184 \\ 176 \\ 216 \\ 109 \\ 45 \end{pmatrix} \\ \hline\hline 1497 \end{array}.$$



```
MATRIX[A]  9 ×9
 .41    .59    0    
 0      0      0    
 .993   0      0    
 0      .987   0    
 0      0      .981 
 0      0      0    
 0      0      0    ↓
1,4=0
```



```
[A]^5*[B]▮
 [[197.4134681]
  [185.444956 ]
  [175.5087689]↑
  [209.0225304]
  [183.7422581]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
▮
```



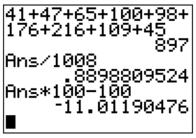
```
197+185+176+209+
184+176+216+109+
45
            1497
Ans/1008
     1.485119048
Ans*100-100▮
```

De groei in 50 jaar is nu nog maar 48,6%.

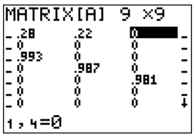
56e

2032

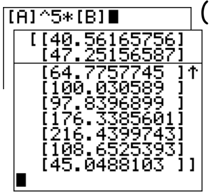
$$(L^{\sim})^5 \cdot P \approx \begin{array}{c} \begin{pmatrix} 41 \\ 47 \\ 65 \\ 100 \\ 98 \\ 176 \\ 216 \\ 109 \\ 45 \end{pmatrix} \\ \hline\hline 897 \end{array}$$



```
41+47+65+100+98+
176+216+109+45
             897
Ans/1008
     .8898809524
Ans*100-100
    -11.01190476
▮
```



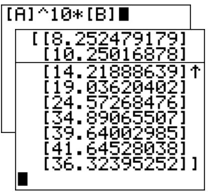
```
MATRIX[A]  9 ×9
 .28    .22    0    
 0      0      0    
 .993   0      0    
 0      .987   0    
 0      0      .981 
 0      0      0    
 0      0      0    ↓
1,4=0
```



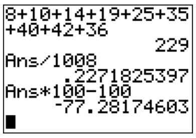
```
[A]^5*[B]▮
 [[40.56165756]
  [47.25156587]
  [64.7757745 ]↑
  [100.030589 ]
  [97.8396899 ]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
▮
```

2082

$$(L^{\sim})^{10} \cdot P \approx \begin{array}{c} \begin{pmatrix} 8 \\ 10 \\ 14 \\ 19 \\ 25 \\ 35 \\ 40 \\ 42 \\ 36 \end{pmatrix} \\ \hline\hline 229 \end{array}.$$



```
[A]^10*[B]▮
 [[8.252479179]
  [10.25016878]
  [14.21888639]↑
  [19.03620402]
  [24.57268476]
  [34.89065507]
  [39.64002985]
  [41.64528038]
  [36.32395252]]
▮
```



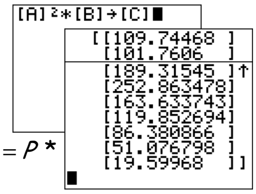
```
8+10+14+19+25+35
+40+42+36
             229
Ans/1008
     .2271825397
Ans*100-100
    -77.28174603
▮
```

Op den duur loopt de bevolking dramatisch terug.

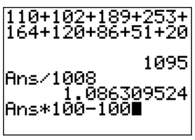
56f

2002

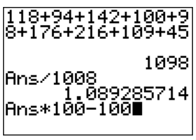
$$(L^{\sim})^2 \cdot P \approx \begin{array}{c} \begin{pmatrix} 110 \\ 102 \\ 189 \\ 253 \\ 164 \\ 120 \\ 86 \\ 51 \\ 20 \end{pmatrix} \\ \hline\hline 1095 \end{array} = P^*$$



```
[A]²*[B]→[C]▮
 [[109.74468 ]
  [101.7606  ]
  [189.31545 ]↑
  [252.863478]
  [163.633743]
  [119.852694]
  [86.380866 ]
  [51.076798 ]
  [19.59968  ]]
▮
```



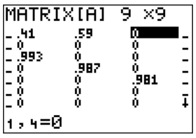
```
110+102+189+253+
164+120+86+51+20

            1095
Ans/1008
     1.086309524
Ans*100-100▮
```



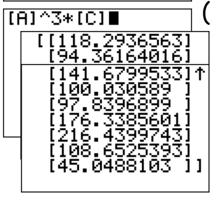
```
118+94+142+100+9
8+176+216+109+45

            1098
Ans/1008
     1.089285714
Ans*100-100▮
```

2032

$$(L^*)^3 \cdot P^* \approx \begin{array}{c} \begin{pmatrix} 118 \\ 94 \\ 142 \\ 100 \\ 98 \\ 176 \\ 216 \\ 109 \\ 45 \end{pmatrix} \\ \hline\hline 1098 \end{array}$$



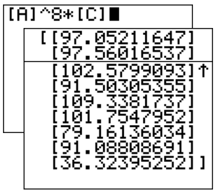
```
MATRIX[A]  9 ×9
 .41    .59    0    
 0      0      0    
 .993   0      0    
 0      .987   0    
 0      0      .981 
 0      0      0    
 0      0      0    ↓
1,4=0
```



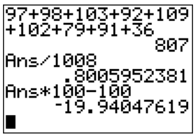
```
[A]^3*[C]▮
 [[118.2936563]
  [94.36164016]
  [141.6799533]↑
  [100.030589 ]
  [97.8396899 ]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
```

2082

$$(L^*)^8 \cdot P^* \approx \begin{array}{c} \begin{pmatrix} 97 \\ 98 \\ 103 \\ 92 \\ 109 \\ 102 \\ 79 \\ 91 \\ 36 \end{pmatrix} \\ \hline\hline 807 \end{array}$$



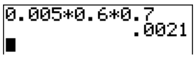
```
[A]^8*[C]▮
 [[97.05211647]
  [97.56016537]
  [102.5799093]↑
  [91.50305355]
  [109.3381737]
  [101.7547952]
  [79.16136034]
  [91.08808691]
  [36.32395252]]
```



```
97+98+103+92+109
+102+79+91+36
             807
Ans/1008
     .8005952381
Ans*100-100
    -19.94047619
▮
```

Op deze manier groeit de bevolking eerst bescheiden en daarna is er een afname.

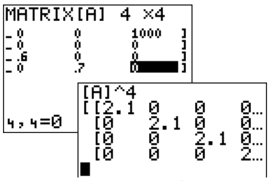
57a Er zijn 4 klassen van elk 2 jaar en $b_{44} = 0 \Rightarrow$ een baars wordt maximaal 8 jaar.
P(eitje groeit tot een baars van 6 jaar) $= 0{,}005 \cdot 0{,}6 \cdot 0{,}7 = 0{,}0021$.



```
0.005*0.6*0.7
            .0021
```

57b

$$B^4 = \text{naar}\;\begin{array}{c|cccc} & \multicolumn{4}{c}{\text{van}} \\ & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 2{,}1 & 0 & 0 & 0 \\ \text{II} & 0 & 2{,}1 & 0 & 0 \\ \text{III} & 0 & 0 & 2{,}1 & 0 \\ \text{IV} & 0 & 0 & 0 & 2{,}1 \end{array}.$$

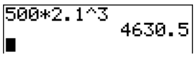


```
MATRIX[A] 4 ×4
[ 0    0    1000 ]
[ 0    0    0    ]
[ .6   0    0    ]
[ 0    .7   0    ]
4,4=0
```

```
[A]^4
[[2.1 0   0   0…
 [0   2.1 0   0…
 [0   0   2.1 0…
 [0   0   0   2…
```

Elke 8 jaar (vier perioden van 2 jaar) wordt de populatie 2,1 ($= 1000 \cdot 0{,}005 \cdot 0{,}6 \cdot 0{,}7$) keer zo groot. Dus de populatie neemt in acht jaar met 110% toe.

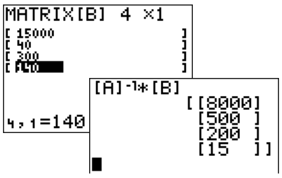
57c Na 24 jaar (3 perioden van 8 jaar) zijn er $500 \cdot 2{,}1^3 \approx 4\,630$ baarzen. (afgerond op tientallen)



```
500*2.1^3
           4630.5
```

57d

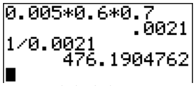
$$B^{-1} \cdot \text{naar}\;\begin{array}{c|c} & \text{1-1-2007} \\ & \text{van totaal} \\ \hline \text{I} & 15000 \\ \text{II} & 40 \\ \text{III} & 300 \\ \text{IV} & 140 \end{array} = \text{naar}\;\begin{array}{c|c} & \text{1-1-2005} \\ & \text{van totaal} \\ \hline \text{I} & 8000 \\ \text{II} & 500 \\ \text{III} & 200 \\ \text{IV} & 15 \end{array}.$$



```
MATRIX[B] 4 ×1
[ 15000 ]
[ 40    ]
[ 300   ]
[ 140   ]
4,1=140
```

```
[A]⁻¹*[B]
        [[8000]
         [500 ]
         [200 ]
         [15  ]]
```

57e Een lagere vruchtbaarheid betekent dat b_{14} kleiner wordt.
Stel dat $b_{14} = v$ dan
$v \cdot 0{,}005 \cdot 0{,}6 \cdot 0{,}7 = 1$ (L^4 is de eenheidsmatrix)
$v \cdot 0{,}0021 = 1$
$v = \frac{1}{0{,}0021} \approx 476.$
Dus een baars legt dan gemiddeld 476 eitjes.



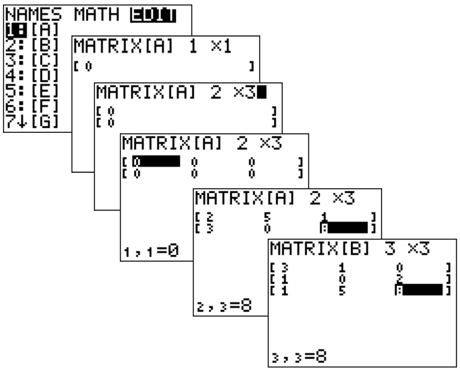
```
0.005*0.6*0.7
            .0021
1/0.0021
      476.1904762
```

TI-84 13. Matrices

De matrix $A = \begin{pmatrix} 2 & 5 & 1 \\ 3 & 0 & 8 \end{pmatrix}$ voer je als volgt op de GR in.

- Kies [MATRIX] (=[2nd][x⁻¹]) en ga met ▶ of ◀ naar het menu EDIT.
- Kies 1: [A] en verander op de bovenste regel de afmeting in 2×3. Na krijg je het vierde scherm hiernaast.
- Tik in [2][ENTER] [5][ENTER] [1][ENTER] [3][ENTER] [0][ENTER] en [8][ENTER]. Je hebt matrix A ingevoerd.

Voer nu ook de matrix $B = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & 2 \\ 1 & 5 & 8 \end{pmatrix}$ op de GR in.



```
NAMES MATH EDIT
1:[A]
2:[B]
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```

```
MATRIX[A] 1 ×1
[ 0 ]
```

```
MATRIX[A] 2 ×3
[ 0 ]
[ 0 ]
```

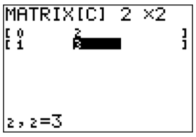
```
MATRIX[A] 2 ×3
[ 0   0   0 ]
[ 0   0   0 ]
1,1=0
```

```
MATRIX[A] 2 ×3
[ 2   5   1 ]
[ 3   0   8 ]
2,3=8
```

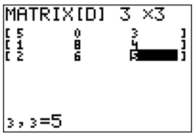
```
MATRIX[B] 3 ×3
[ 3   1   0 ]
[ 1   0   2 ]
[ 1   5   8 ]
3,3=8
```

Heb je een fout gemaakt bij het invoeren, dan ga je met de cursor naar de fout, waarna de fout te herstellen is.

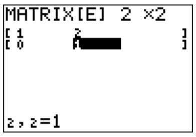
🖩1a Zie de schermen hiernaast.
🖩1b 10.
🖩1c *
🖩1d *



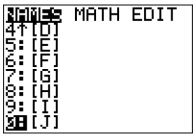
```
MATRIX[C] 2 ×2
[ 0   2 ]
[ 1   3 ]
2,2=3
```



```
MATRIX[D] 3 ×3
[ 5   0   3 ]
[ 1   8   4 ]
[ 2   6   5 ]
3,3=5
```

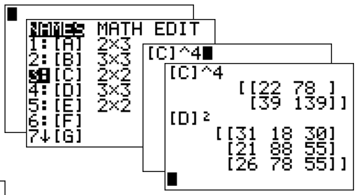


```
MATRIX[E] 2 ×2
[ 1   2 ]
[ 0   1 ]
2,2=1
```



```
NAMES MATH EDIT
4↑[D]
5:[E]
6:[F]
7:[G]
8:[H]
9:[I]
0:[J]
```

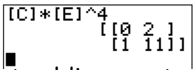
🖩2a $C^4 = \begin{pmatrix} 22 & 78 \\ 39 & 139 \end{pmatrix}$ en $D^2 = \begin{pmatrix} 31 & 18 & 30 \\ 21 & 88 & 55 \\ 26 & 78 & 55 \end{pmatrix}$.



```
NAMES MATH EDIT
1:[A]  2×3
2:[B]  3×3
3:[C]  2×2
4:[D]  3×3
5:[E]  2×2
6:[F]
7↓[G]
```

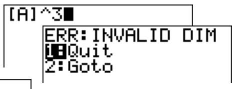
```
[C]^4
        [[22 78 ]
         [39 139]]
[D]²
     [[31 18 30]
      [21 88 55]
      [26 78 55]]
```

🖩2b $C \cdot E^4 = \begin{pmatrix} 0 & 2 \\ 1 & 11 \end{pmatrix}$.



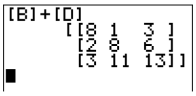
```
[C]*[E]^4
          [[0 2 ]
           [1 11]]
```

🖩2c A^3 geeft een foutmelding, want A is geen vierkante matrix.



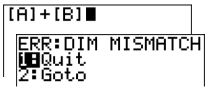
```
[A]^3
ERR:INVALID DIM
1:Quit
2:Goto
```

🖩3a $B + D = \begin{pmatrix} 8 & 1 & 3 \\ 2 & 8 & 6 \\ 3 & 11 & 13 \end{pmatrix}$.



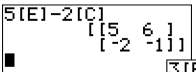
```
[B]+[D]
     [[8  1  3 ]
      [2  8  6 ]
      [3  11 13]]
```

🖩3b $A + B$ geeft een foutmelding, want A en B hebben niet dezelfde afmeting.



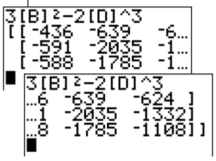
```
[A]+[B]
ERR:DIM MISMATCH
1:Quit
2:Goto
```

🖩3c $5E - 2C = \begin{pmatrix} 5 & 6 \\ -2 & -1 \end{pmatrix}$.



```
5[E]-2[C]
          [[5  6 ]
           [-2 -1]]
```

🖩3d $3B^2 - 2D^3 = \begin{pmatrix} -436 & -639 & -624 \\ -591 & -2035 & -1332 \\ -588 & -1785 & -1108 \end{pmatrix}$.



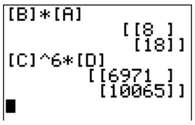
```
3[B]²-2[D]^3
[[-436 -639  -6…
 [-591 -2035 -1…
 [-588 -1785 -1…
```

```
3[B]²-2[D]^3
…6 -639  -624 ]
…1 -2035 -1332]
…8 -1785 -1108]]
```

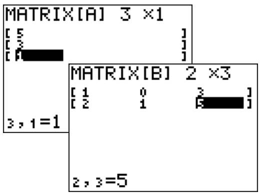
🖩4a Zie de schermen hiernaast.

🖩4b $B \cdot A = \begin{pmatrix} 8 \\ 18 \end{pmatrix}$.

🖩4c $C^6 \cdot D = \begin{pmatrix} 6971 \\ 10065 \end{pmatrix}$.

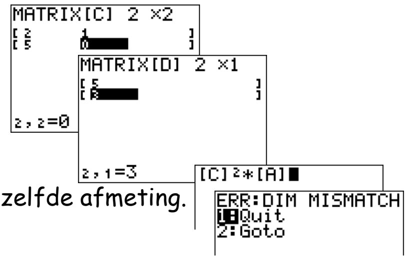


```
[B]*[A]
         [[8 ]
          [18]]
[C]^6*[D]
      [[6971 ]
       [10065]]
```



```
MATRIX[A] 3 ×1
[ 5 ]
[ 3 ]
[ 1 ]
3,1=1
```

```
MATRIX[B] 2 ×3
[ 1   0   3 ]
[ 2   1   5 ]
2,3=5
```



```
MATRIX[C] 2 ×2
[ 2   1 ]
[ 5   0 ]
2,2=0
```

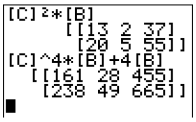
```
MATRIX[D] 2 ×1
[ 5 ]
[ 3 ]
2,1=3
```

```
[C]²*[A]
ERR:DIM MISMATCH
1:Quit
2:Goto
```

🖩4d $C^2 \cdot A$ geeft een foutmelding want A en B hebben niet dezelfde afmeting.

$C^2 \cdot B = \begin{pmatrix} 13 & 2 & 37 \\ 20 & 5 & 55 \end{pmatrix}$.

$C^4 \cdot B + 4B = \begin{pmatrix} 161 & 28 & 455 \\ 238 & 49 & 655 \end{pmatrix}$.



```
[C]²*[B]
      [[13 2 37]
       [20 5 55]]
[C]^4*[B]+4[B]
   [[161 28 455]
    [238 49 665]]
```